TECHNICAL SCIENCES AND TECHNOLOGIES
UDC 519.168:004.94
DOI: 10.25140/2411-5363-2018-3(13)-89-95

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ALGORITHM OF EULER-LAGRANGE METHOD FOR DEISGNING OF DYNAMIC MODEL

Urgency of the research. *Nowadays robotics and mechatronics come to be mainstream. With development in these areas also grow computing fastidiousness. Since there is significant focus on numerical modeling and algorithmization in kinematic and dynamic modeling.*

Target setting. *By automation of whole process of dynamic model design the errors are eliminated as well as the time of designing significantly decreases.*

Actual scientific researches and issues analysis. *Designing of dynamic model by analytical way is very difficult especially in the cases considering high number of DOF. For hyper-redundant manipulators it is practically impossible. From this reason whole process is automatized.*

Uninvestigated parts of general matters defining. *The theory of Euler – Lagrange method is automatized by means of robotic view on this issue.*

The research objective. *In the paper, an algorithm for design of dynamic model was introduced.*

The statement of basic materials. *The paper deals with automatic design process of dynamic model for serial kinematic structure mechanisms. In the paper Euler – Lagrange formula is discussed. Analytical way of dynamic modeling should be difficult problem especially for mechanisms with high number of degrees of freedom. From this reason the paper shows the way of automatically designing of dynamic modeling in MATLAB. Our study shows dependence of computing time on increasing DOF. The relation is expressed by function of 3rd order. Subsequently the paper presents automatically generated inverse dynamic model in cooperation with inverse kinematic model as well as trajectory planning task.*

Conclusions. *The paper introduces automatically generated dynamic model for mechanisms with serial kinematic structure. The paper also established the time for designing of dynamic model for several mechanisms with changing DOF.*

Keywords: *dynamic model; Euler–Lagrange; kinematically redundant mechanism.*

Fig.: 7. References: 7.

Introduction. In general, dynamic model describes the relationship between force (torques) and motion of investigated mechanism. Mathematical – dynamic model – is important part of designing of mechanisms and robots in order to its motion simulations, optimization as well as control algorithm design [1]. Many prior works have been done in the research field of dynamic modeling [2]-[4]. A researchers working in the area of robotics are also focused on the computational efficiency of robot control systems [5][6]. For robot dynamic description two main methods are usually used, namely Newton – Euler method and Euler – Lagrange method [7]. This paper deals with the second of them. The aim of the paper is to implement theory of Euler – Lagrange method in computing algorithm for simplification of dynamic model designing especially for mechanisms with high number of DOF.

The paper is divided as follows. The 2nd chapter deals with Euler – Lagrange method. The 3rd chapter presents the algorithm for dynamic model design. This process is numerically simulated for nine mechanisms with different DOF. Next, the relation between computing time of simulation and increasing number of DOF is presented. Subsequently, following chapter shows the example of automatically designed dynamic model on planar mechanism with 6 DOF.

Dynamic model – Euler – Lagrange Method. In consideration of dynamic model there are two basic problems, namely forward and inverse model. The forward dynamic model computes the joint accelerations and others kinematic variables like velocity and position, while generalized forces (torques) are given. The inverse dynamic model computes of forces (torques), while kinematic variables (acceleration, velocity, position) are known.

Both methods, Newton – Euler as well as Euler - Lagrange have different base. Nevertheless, both methods result in the same generalized dynamic equation

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}\,, \tag{1}$$

where $\mathbf{M}$ is matrix of inertia $\mathbf{M} \in \mathbb{R}^{n\,x\,n}$, $\mathbf{C}$ is matrix of Coriolis and centrifugal forces $\mathbf{C} \in \mathbb{R}^{n\,x\,n}$, $\mathbf{D}$ is diagonal matrix considering friction $\mathbf{D} \in \mathbb{R}^{n\,x\,n}$, $\mathbf{g}$ is vector of gravity forces $\mathbf{g} \in \mathbb{R}^{n}$, $\boldsymbol{\tau}$ is vector of torques $\boldsymbol{\tau} \in \mathbb{R}^{n}$ and $\mathbf{q} \in \mathbb{R}^{n}$ is vector of generalized variables. Parameter *n* represents number of DOF of investigated mechanism.

In the following section introduces the theory of Euler – Lagrange method. As have been mentioned above, the method arises from kinetic and potential energy what by equation (2) is expressed

$$\psi_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} \tag{2}$$

where ψ_i represents extern and dissipative generalized forces. The function $L(q,\ \dot{q}) = K(q,\ \dot{q}) - P(q)$ represents Lagrange function. The equation (2) is

$$\psi_i = \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_i}\right) - \frac{\partial K}{\partial q_i} + \frac{\partial P}{\partial q_i} \tag{3}$$

The final kinetic and potential energy of investigated mechanism is expressed by sum of kinetic and potential energy of all mechanism links

$$K = \sum_{i=1}^{n} K_i \tag{4}$$

$$P = \sum_{i=1}^{n} P_i \tag{5}$$

Kinetic energy of *i*-th link can be expressed as follows

$$K_i = \frac{1}{2} m_i \mathbf{v}_{ci}^T \mathbf{v}_{ci} + \frac{1}{2} \omega_i^T \mathbf{R}_i \mathbf{I}_i \mathbf{R}_i^T \omega_i \tag{6}$$

$$K_i = \frac{1}{2} \dot{\mathbf{q}}^T \sum_{i=1}^{n} \left[m_i \mathbf{J}_v^{i\,T}(\mathbf{q}) \mathbf{J}_v^i(\mathbf{q}) + \mathbf{J}_{\boldsymbol{\omega}}^{i\,T}(\mathbf{q}) \mathbf{R}_i \mathbf{I}_i \mathbf{R}_i^T \mathbf{J}_{\boldsymbol{\omega}}^i(\mathbf{q}) \right] \dot{\mathbf{q}} \tag{7}$$

$$K_i = \frac{1}{2} m_i \mathbf{v}_{ci}^T \mathbf{v}_{ci} + \frac{1}{2} \omega_i^T \mathbf{R}_i \mathbf{I}_i \mathbf{R}_i^T \omega_i \tag{8}$$

$$K_i = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} M_{ij}(\mathbf{q}) \dot{q}_i \dot{q}_j \tag{9}$$

where $\mathbf{R}_i$ is rotational matrix between fixed frame of mechanism and *i*-th link, $\mathbf{I}_i$is matrix of moments of inertia, $\mathbf{J}_v^i$ and $\mathbf{J}_{\boldsymbol{\omega}}^i$ are Jacobians for linear and revolute velocity.

Potential energy of *i*-th link can be expressed as follows

$$P_i = \int_{L_i} \mathbf{g}^T \mathbf{p}\, dm = \mathbf{g}^T \int_{L_i} \mathbf{p}\, dm = \mathbf{g}^T \mathbf{p}_{ci} m_i \tag{10}$$

where $= [0 \quad -9.81 \quad 0]^T$. Total potential energy is now

$$P = \sum_{i=1}^{n} \mathbf{g}^T \mathbf{p}_{ci} m_i \tag{11}$$

Lagrange function is then

$$L = K - P = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} M_{ij}(\mathbf{q}) \dot{q}_i \dot{q}_j - \sum_{i=1}^{n} \mathbf{g}^T \mathbf{p}_{ci} m_i \tag{12}$$

$$\frac{\partial L}{\partial \dot{q}_k} = \frac{\partial K}{\partial \dot{q}_k} = \sum_{j=1}^{n} M_{kj} \dot{q}_j \tag{13}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} = \sum_{j=1}^{n} M_{kj} \ddot{q}_j + \sum_{j=1}^{n} \frac{dM_{kj}}{dt} \dot{q}_j = \sum_{j=1}^{n} M_{kj} \ddot{q}_j + \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial M_{kj}}{\partial q_i} \dot{q}_i \dot{q}_j \tag{14}$$

$$\frac{\partial L}{\partial q_k} = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial M_{ij}}{\partial q_k} \dot{q}_i \dot{q}_j - \frac{\partial P}{\partial q_k} \tag{15}$$

$$\sum_{j=1}^{n} M_{kj} \ddot{q}_j + \sum_{i=1}^{n} \sum_{j=1}^{n} \left[\frac{\partial M_{kj}}{\partial q_i} - \frac{1}{2} \frac{\partial M_{ij}}{\partial q_k} \right] \dot{q}_i \dot{q}_j + \frac{\partial P}{\partial q_k} = \psi_k \tag{16}$$

By following substitution

$$h_{kji}(\mathbf{q}) = \frac{\partial M_{kj}(\mathbf{q})}{\partial q_i} - \frac{1}{2} \frac{\partial M_{ij}(\mathbf{q})}{\partial q_k} \tag{17}$$

$$g_k(\mathbf{q}) = \frac{\partial P(\mathbf{q})}{\partial q_k} \tag{18}$$

one obtains

$$\sum_{j=1}^{n} M_{kj}(\mathbf{q}) \ddot{q}_j + \sum_{i=1}^{n} \sum_{j=1}^{n} h_{kji}(\mathbf{q}) \dot{q}_i \dot{q}_j + g_k(\mathbf{q}) = \psi_k \tag{19}$$

Equation (19) can be now written in the form

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau} \tag{20}$$

where matrix $\mathbf{C}$ can be expressed as

$$C_{kj} = \sum_{i=1}^{n} c_{ijk} \dot{q}_i \tag{21}$$

$$c_{ijk} = \frac{1}{2}\left[\frac{\partial M_{kj}}{\partial q_i} + \frac{\partial M_{ki}}{\partial q_j} - \frac{\partial M_{ij}}{\partial q_k}\right] \tag{22}$$

The parameters in the equation (22) are so called Christoffel symbols.

Algorithm of Automatic Designing of Dynamic model. The increasing number of DOF significantly increases the fastidiousness of dynamic model designing by analytical way. With consideration of kinematically redundant mechanism the inclination to the dynamic model designing failure is too high. Therefore it is desirable to design system for automatic designing of dynamic model. The contribution of this paper is to introduce the algorithm which simplify dynamic model designing.

Introduced algorithm considers the mechanisms with serial kinematic structure so called open chain mechanism. It could be used for planar as well as three-dimensional mechanism.

Algorithm: Design of Dynamic Model

1: Definition of basic parameters of links like weight *m,* length *L*, number of links *N*. Definition of kind of friction and friction coefficients in the joints, definition of rotational matrices $\mathbf{R}_i$
2: Computation of COG (center of gravity) positions for particular links from the fixed base
3: Computation of Jacobians for linear $\mathbf{J}_v^i$ and revolute $\mathbf{J}_\omega^i$ motions
4: Computation of potential energy for particular links
5: Computation of vector of gravity forces by relations $\boldsymbol{P} = \sum_{i=1}^{n} \mathbf{g}^T \mathbf{p}_{ci} \boldsymbol{m}_i$ and $\frac{\partial P(\mathrm{q})}{\partial q_k}$
6: Computation of inertia matrix by relation $\sum_{i=1}^{n}\left[\boldsymbol{m}_i \mathbf{J}_v^{i\,T}(\mathbf{q})\mathbf{J}_v^i(\mathbf{q}) + \mathbf{J}_\omega^{i\,T}(\mathbf{q})\mathbf{R}_i\mathbf{I}_i\mathbf{R}_i^T\mathbf{J}_\omega^i(\mathbf{q})\right]$
7: Computation of matrix of Coriolis and centrifugal forces by cycles FOR

FOR *k*=1 TO *N*
8: **FOR** *j*=1 TO *N*
9: **FOR** *i*=1 TO *N*
10: $c_{ijk} = \frac{1}{2}\left[\frac{\partial M_{kj}}{\partial q_i} + \frac{\partial M_{ki}}{\partial q_j} - \frac{\partial M_{ij}}{\partial q_k}\right]$
11: **END FOR**
12: $C_{kj} = \sum_{i=1}^{n} c_{ijk}\dot{q}_i$
13: c_{ijk}
14: **END FOR**
15: END FOR

When basic parameters are defined in the step 1, the output of this point is the matrix of inertia $\mathbf{M}(\mathbf{q})$, matrix of Coriolis and centrifugal forces $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ and vector of gravity forces $\mathbf{g}(\mathbf{q})$. Matrix of friction forces if manually given by user according to kind of friction in the joints. The advantage of the algorithm is generalization and therefore doesn´t how many links mechanism has. User only sets the number of links.

Mentioned algorithm will be now tested for mechanisms with different number of DOF. This section shows how the fastidiousness of dynamic model designing increases with increasing number of DOF. The simulation is focused on computing time of designing of dynamic model in dependence on increasing number of mechanism DOF.

The parameters of testing PC is Intel Core i3-4000M CPU 2.40 GHz, RAM 4,00 GB, OS: Windows 7 x64. The algorithm has been run in software MATLAB 2013a. Considering each DOF for mechanism, the simulation has been performed 10 times for each DOF. Final computing time was stated as average value of all 10 simulations for each DOF. The results of all simulations for all investigated mechanisms in the Figure 1 are shown.

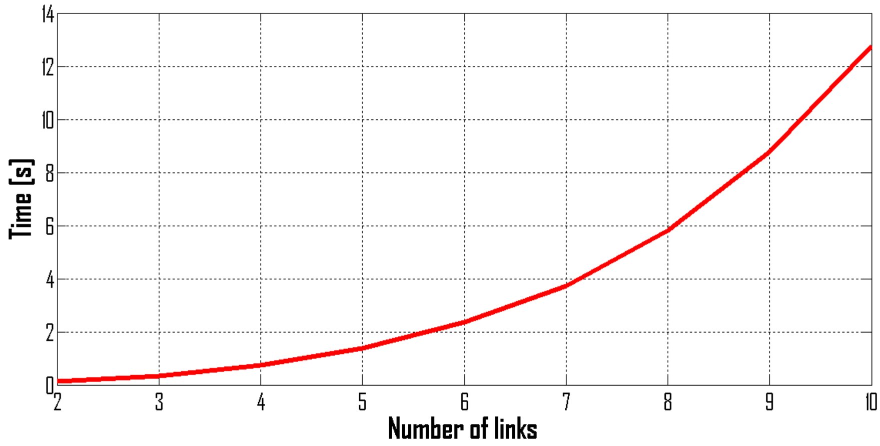

Fig. 1. Increasing computing time for dynamic model design in dependence on increasing DOF of mechanism

The function of increasing time in dependence on increasing DOF of mechanism can be also expressed by polynomial function of 3rd order

$$y = 0{,}0191x^3 - 0{,}1025x^2 + 0{,}3757x - 0{,}3647. \quad (23)$$

Let´s now consider kinematically hyper-redundant mechanism what can be for example continuum robot. Considering the equation (23) as well as the same PC parameters, the computing time for mechanism with 30 links 434,3563 s and for mechanism with 100 links 18112,206 s. It has to be also noticed the impossibility of human designing of dynamic model for mentioned mechanisms.

Example of algorithm utilization for inverse dynamics.

This section deals with study case of dynamic model designing by introduced algorithm with consideration of inverse kinematic model and trajectory planning path of kinematically redundant mechanism. Within this study will be investigated planar mechanism.

The aim of following task is finding generalized forces (torques) in order to move end-effector from its start position to the goal position. Input to the system is point $\mathbf{P} \in \mathbb{R}^m$. Point $\mathbf{P}$ are the points of required end-effector position at the end of the motion.

At first, we need to use inverse kinematic model for computation of final positions of mechanism joints $\mathbf{q} \in \mathbb{R}^n$. Next, the trajectory has to be planned for particular links. For this operation will be used polynomial of 5th order. Polynomial of 3rd order withholds opportunity to set initial and goal conditions for acceleration / deceleration of particular links during its motion.

The output of trajectory planning task is matrix of angular positions $\in \mathbb{R}^{t\,x\,n}$, matrix of angular velocities $\dot{\mathbf{Q}} \in \mathbb{R}^{t\,x\,n}$ and matrix of angular accelerations $\ddot{\mathbf{Q}} \in \mathbb{R}^{t\,x\,n}$. Parameter t is number of points which are placed between initial and goal position of end-effector. The output this task is the input to the inverse dynamic model block, see Figure 2. The output of inverse dynamic model block are torques $\boldsymbol{\tau} \in \mathbb{R}^n$ in the vector form.

Fig. 2. Flow of task

The Figure 3 shows start and goal configuration of links of investigated mechanism with 6 DOF. Initial angle for all joints is 0^0. The goal position of end-effector is point [1, 2]. Length of all links is L = 1 m.

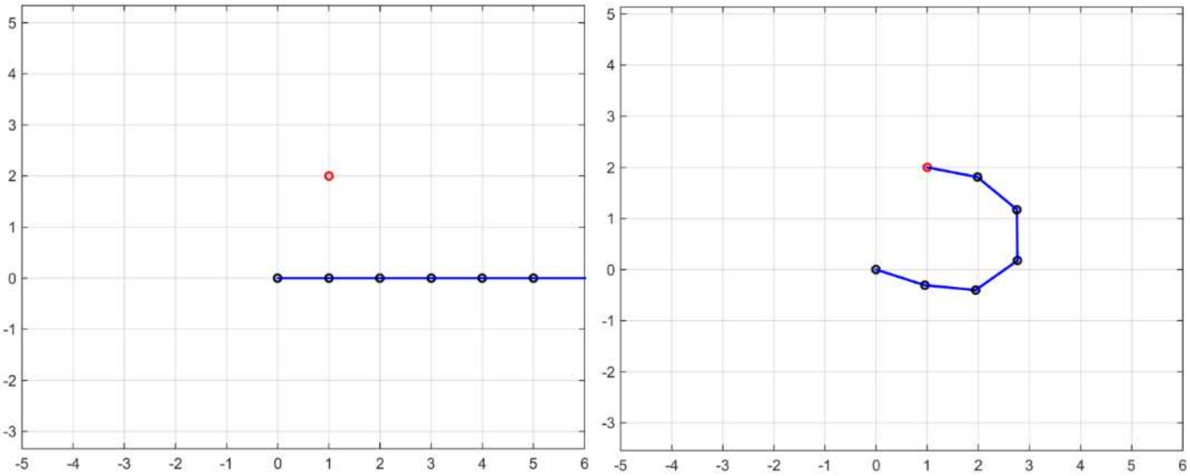

Fig. 3. Start and goal links configuration of mechanism

Damped least squares method is used for inverse kinematic model computation. The result of inverse kinematic model is vector of generalized variables **q**. The Figure 4, Figure 5 and Figure 6 show the result of trajectory planning block. AS have been mentioned, the trajectory planner used polynomial function of 5th order.

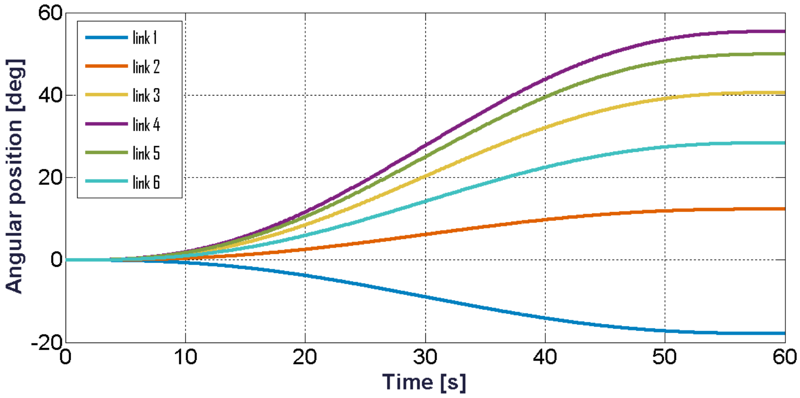

Fig. 4. Trajectory planning - Angular positions

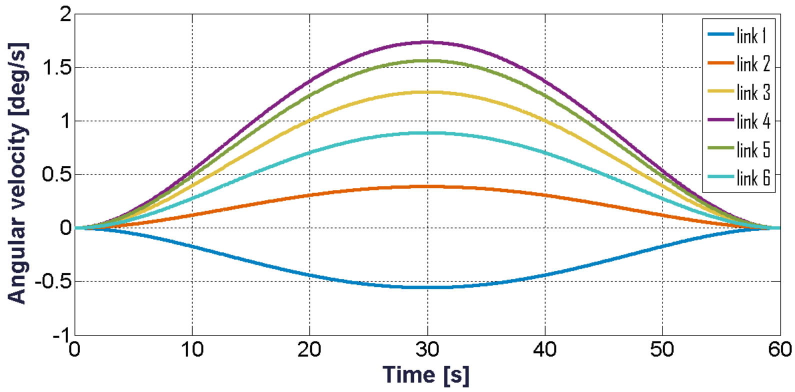

Fig. 5. Trajectory planning - Angular velocities

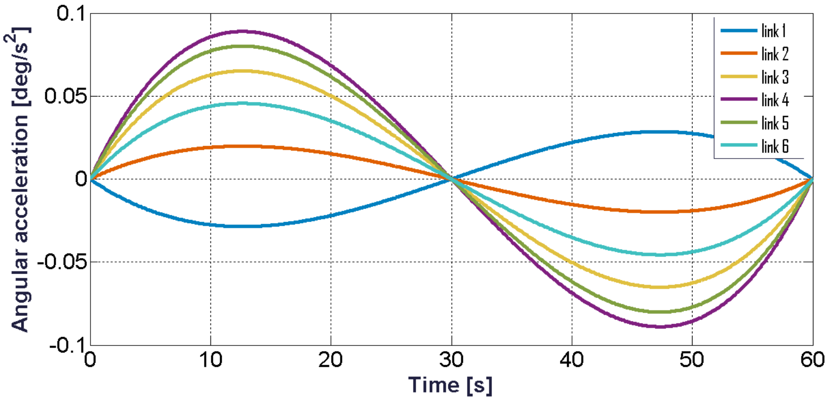

Fig. 6. Trajectory planning - Angular accelerations

As can be seen from the Figures 4, 5 and 6, the simulation assumed zero initial and goal conditions. According to Fig. 2, the output is inverse dynamics what is in our case vector of required torques, see Figure 7.

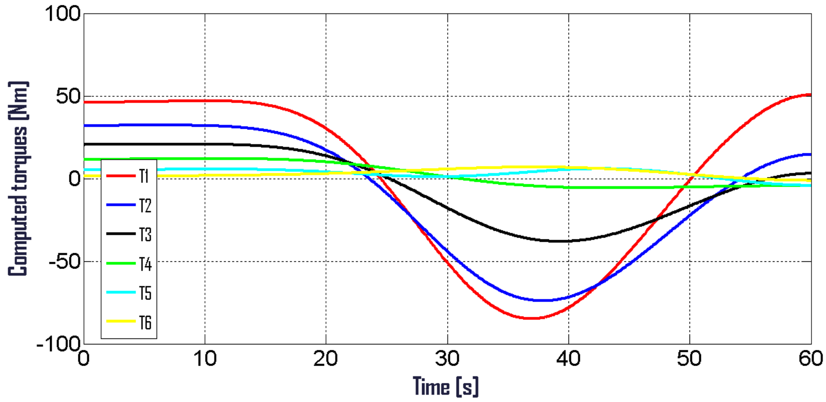

Fig. 7. Result of the simulation - Required torques

The simulation lasts 60 s. Of course, the lower time of the simulation would be, the higher torques would be required in order to achievement of required vector of generalized variables and its derivations.

Conclusion. The paper dealt with one of two basic approaches for dynamic model designing Euler – Lagrange method. The algorithm for automatic designing of dynamic model is developed and numerically tested. The algorithm is programmed in MATLAB and tested on planar mechanisms with six degrees of freedom. Also the paper states the dependence of increasing computing time on increasing DOF of mechanism. The contribution of the paper in the form of computational algorithm is tool for engineering research. Designed dynamic model by this algorithm can be then used for optimization and control task.

Acknowledgement. The author would like to thank to Slovak Grant Agency – project VEGA 1/0389/18.

References

1. Spong, M. W., Hutchinson, S. & Vidyasagar, M., (2006). *Robot Modeling and Control, ISBN: 978-0-471-64990-8.* JOHN WILEY & SONS, INC [in English].

2. Featherstone, R. (1983). The Calculation of Robot Dynamics using Articulated-Body Inertias. *International Journal of Robotics Research, 1(2*), 13–30 [in English].

3. J. Y. S. Luh, M. W. Walker and R. P. C. Paul (1980). On-Line Computational Scheme for Mechanical Manipulators. (J. ASME, Trans.). *Dynamic Systems, Measurement & Control, 2(102*), 69–76 [in English].

4. Walker, M. W. & Orin, D. E. (1982). Efficient Dynamic Computer Simulation of Robotic Mechanisms. (J. ASME, Trans.). *Dynamic Systems, Measurement & Control, (104*), 205-211 [in English].

5. Featherstone, R. & Orin, D. (2000). Robot Dynamics: Equations and Algorithms. Proceedings from: *IEEE International Conference on Robotics & Automation* (pp. 826 – 834) [in English].

6. Sciavicco, L. & Siciliano, B. (2000). *Modelling and control of robot manipulators.* Springer Verlag [in English].

7. Asada, H. H., (2005). *Introduction to Robotics*. Massachusetts Institute of Technology [in English].

УДК 519.168:004.94

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АЛГОРИТМ ЕЙЛЕРА-ЛАГРАНЖА ДЛЯ ПРОЕКТУВАННЯ ДИНАМІЧНОЇ МОДЕЛІ

Актуальність теми дослідження. *В наші дні робототехніка і мехатроніка стають мейнстримом. З розвитком цих галузей також зростають вимоги до обчислювальної техніки, оскільки основна увага приділяється чисельному моделюванню й алгоритмізації в кінематичному та динамічному моделюванні.*

Постановка проблеми. *При автоматизації всього процесу проектування динамічної моделі усуваються помилки, а час проектування значно зменшується.*

Аналіз останніх досліджень і публікацій. *Проектування динамічної моделі аналітичним способом дуже складний процес, особливо у випадках урахування глибини зображуваного простору. Для гіпернадлишкових маніпуляторів це практично неможливо. З цієї причини весь процес автоматизований.*

Виділення недосліджених частин загальної проблеми. *Теорія методу Ейлера-Лагранжа автоматизована за допомогою роботизованого погляду на це питання.*

Постановка завдання. *У статті був запропонований алгоритм проектування динамічної моделі.*

Виклад основного матеріалу. *У статті розглядається процес автоматизованого проектування динамічної моделі механізмів з послідовною кінематичної структурою. У статті обговорюється формула Ейлера - Лагранжа. Аналітичний спосіб динамічного моделювання завжди є складною проблемою, особливо для механізмів з великою кількістю ступенів свободи. З цієї причини в статті показаний спосіб автоматизованого проектування динамічної моделі в MATLAB. Наше дослідження показує залежність часу обчислень від збільшення ступеня свободи. Залежність є функцією 3-го порядку. Насамкінець в роботі представлена автоматично створена зворотна динамічна модель в комплексі зі зворотною кінематичною моделлю, а також завдання планування траєкторії.*

Висновки відповідно до статті. *У статті наведена автоматизовано створена динамічна модель для механізмів із послідовною кінематичної структурою. У роботі також було визначено час для розробки динамічної моделі для декількох механізмів зі змінним числом ступенів волі.*

Ключові слова: *динамічна модель; Ейлер-Лагранж; кінематично-надлишковий механізм.*

Рис.: 7. Бібл.: 7.

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Virgala, I., Filakovský, F. (2018). Algorithm of euler-lagrange method for deisgning of dynamic model. *Technical sciences and technologies*, *3* (13), 89–95.