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BOND GRAPH METHODOLOGY IN THE RLC CIRCUIT ANALYSIS

Urgency of the research. *The bond graphs theory aim for to formulate general class physical systems over power interactions. The factors of power are effort and flow. They have different interpretations in different physical domains. Yet, power can always be used as a generalized resource to model coupled systems residing in several energy domains.*

Target setting. *Formalism of power graphs enables to describe different physical systems and their interactions in a uniform, algorithmizable way and transform them into state space description. This is useful when analyzing mechatronic systems transforming various forms of energy (electrical, fluid, mechanical) by means of information signals to the resulting mechanical energy.*

Actual scientific researches and issues analysis. *Over the past two decades the theory of Bond Graphs has been paying attention to universities around the world, and bond graphs have been part of study programs at an ever-increasing number of universities. In the last decade, their industrial use is becoming increasingly important. The Bond Graphs method was introduced by Henry M. Paynter (1923-2002), a professor at MIT & UT Austin, who started publishing his works since 1959 and gradually worked out the terminology and formalism known today as Bond Graphs translated as binding graphs or performance graphs.*

Uninvestigated parts of general matters defining. *The electrical system model is solved with the help of the above mentioned bond graphs formalism. Gradually, the theory of power graphs in the above example is explained up to the construction of state equations of the electrical system. The state equations are then solved in Matlab / Simulink.*

The statement of basic materials. *Using bond graphs theory to simulate electrical system and verify its suitability for simulating electrical models. In various versions of the parameters of model we can monitor its behavior under different operating conditions. The language of bond graphs aspires to express general class physical systems through power interactions. The factors of power i.e., effort and flow, have different interpretations in different physical domains. Yet, power can always be used as a generalized coordinate to model coupled systems residing in several energy domains.*

Conclusions. *We introduced a method of systematically constructing a bond graph of an electrical system model using Bond graphs. A practical example of an electrical model is given as an application of this methodology. Causal analysis also provides information about the correctness of the model. Differential equations describing the dynamics of the system in terms of system states were derived from a simple electrical system coupling graph. The results correspond to the equations obtained by the classical manual method, where first the equations for individual components are created and then a simulation scheme is derived based on them. The presented methodology uses the reverse procedure. However, manually deriving equations for more complex systems is not so simple. Bond charts prove to be a suitable means of analysis, among other systems and electrical systems.*

Keywords: *mechatronics; energy modeling; Bond Graphs; modeling of dynamic system; source of an effort; source of an flow; capacitor, inductor; resistor.*
Fig.: 4. References: 18.

Introduction. The aim of this paper is the interest of the use of bond graphs as modeling tool. Bond graph is an explicit graphical tool for capturing the common energy structure of systems. The work shows the use of Bond Graph formalism for modeling dynamic systems. In contrast with the classical method, where the equations for individual components are created first and then the simulation scheme is derived on their basis, the described method uses the reverse procedure. As an example an electrical RLC circuit system is solved by this approach at the level of its physical behavior. In this paper the method of generation of system equations is discussed. From a bond graph diagram of the system, using a step-by-step procedure, system equations may be generated. The differential equations describing the dynamics of the system are obtained in terms of the states of the system.

The concept of bond graphs was originated by Paynter (1961). The idea was further developed by Karnopp and Rosenberg in their textbooks (1968, 1975, 1983, 1990), such that it could be used in practice (Thoma, 1975; Van Dixhoorn, 1982). By means of the formulation by Breedveld (1984, 1985) of a framework based on thermodynamics, bond-graph model description evolved to a systems theory. The language of bond graphs aspires to express general class physical systems through power interactions. The factors of power i.e., effort and flow, have different interpretations in different physical domains. Yet, power can always be used as a generalized co-ordinate to model coupled systems residing in several energy domains [1-4].

Bond graphs theory. A subsystem is represented by a closed line with a name. This line represents the frontiers of the subsystem. For each energy interchange of the system with its environment we associate to it an energetic port of a defined type (mechanical energy, electrical energy, etc.). A unidirectional semi headed arrow shows the energy interchange throw this port and carries the data relative to the power transported (e: effort and f: flow). These two

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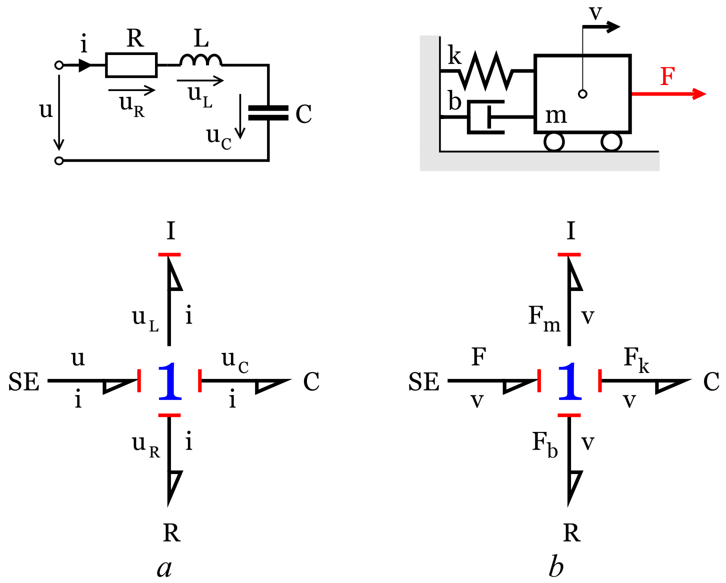
variables are necessary and sufficient to describe the energetic transfers inside the system. They correspond to a couple of variables in each energetic domain. The elementary components/subsystems are classified by their energetic behaviour (energy dissipation, energy storage, etc.), by their function inside the system (flows sensor, etc.).

Bond Graph Standard Elements. In bond graphs, one needs to recognize only four groups of basic symbols, i.e., three basic one port passive elements, two basic active elements, two basic two port elements and two basic junctions. The basic variables are effort (e), flow (f), time integral of effort (h) and time integral of flow (q). Examples of mechanical and electrical systems in bond graphs methodology are shown in Fig. 1.



a *b*

Fig. 1. Physical system:
a –electrical system RLC circuit; b – mechanical system mass-spring-damper

Source: [6; 7].

Effort and flow sources. The active ports are those, which give reaction to the source. For, example if we step on a rigid body, our feet reacts with a force or source. For this reason, sources are called active ports. Force is considered as an effort source and the surface of a rigid body gives a velocity source. They are represented as a half arrow pointing away from the source symbol. In electrical domain, an ideal shell would be represented as an effort source. Similarities can be drawn for source representations in other domains [5-7].

Description of the model. To demonstrate the bond graph methodology as an example an electrical model of RLC system is analyzed Fig. 2. An RLC circuit (or LCR circuit) is an electrical circuit consisting of a resistor, an inductor, and a capacitor. The RLC part of the name is due to those letters being the usual electrical symbols for resistance, inductance and capacitance respectively [8-12].

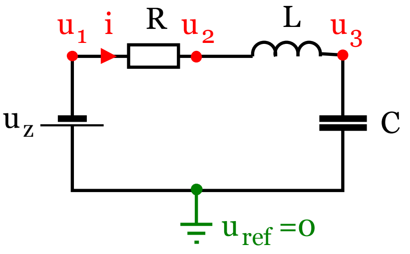


Fig. 2. Electrical system RLC circuit with reference voltage u_{ref}, voltages u_1, u_2, u_3 of the element ports

Source: [6; 7].

Systematic procedure to derive a bond graph. We have discussed the basic bond-graph elements and the bond, so we can transform a domain-dependent ideal-physical model, written in domain-dependent symbols, into a bond graph. For this transformation, there is a systematic procedure, which is presented here [6, 7, 12]. First we determine which physical domains exist in the system and identify all basic elements like C (capacitor), I (inductor), R (rezistor), SE (source of the effort), SF (source of the flow), TF (transformer) and GY (gyrator). This system contains a electrical domain part with the inductance L of the inductor (I:L), the resistance R of the resistor (R:R) and a capacity of the condensator C (C:C). Voltage u_z is considered as an effort source (SE: u_z). Indicate in the ideal-physical model per domain a reference source – effort voltage u_{ref} (reference voltage with positive direction), Fig. 2. The references are indicated in the ideal physical model: the ideal velocity $u_{ref} = 0$. Identify all other efforts (voltages) and give them unique names u_1, u_2, u_3, Fig. 2. Draw these efforts (electrical: voltages), graphically by 0-junctions in Fig. 3.

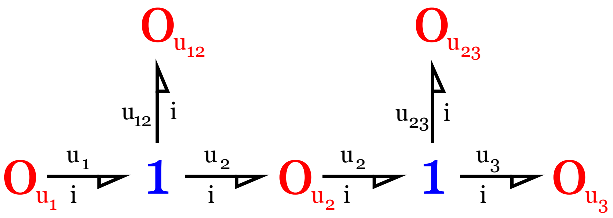


Fig. 3. Voltages as 0-junctions and connecting 0-junctions with 1-junctions

Source: [6; 7].

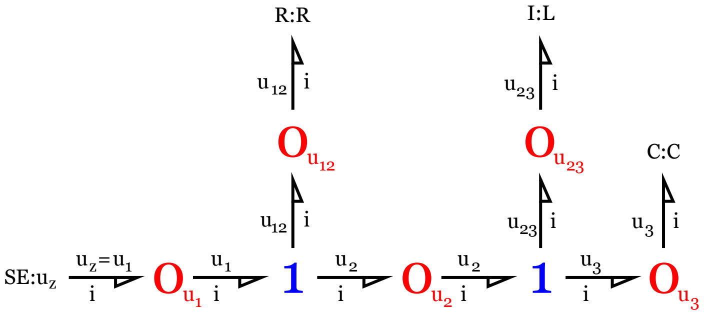


Fig. 4. Connecting elements R, I, C, SE with 0-junctions

Source: [6; 7].

Identify all effort differences (electrical: voltage = effort) needed to connect the ports of all elements enumerated in Fig. 3 to the junction structure. When checking all ports of the elements found the linear voltage differences, u_{12} and u_{23} are identified. Construct the voltages differences using a 1-junction and draw them as such in the graph Fig. 3. The junction structure is now ready and the elements can be connected. Connect the port of all elements found at step 1 with the 1-junction of the corresponding efforts or efforts differences Fig. 4. Simplify the resulting graph by applying the simplification rules in Fig. 5 and Fig. 6 [7].

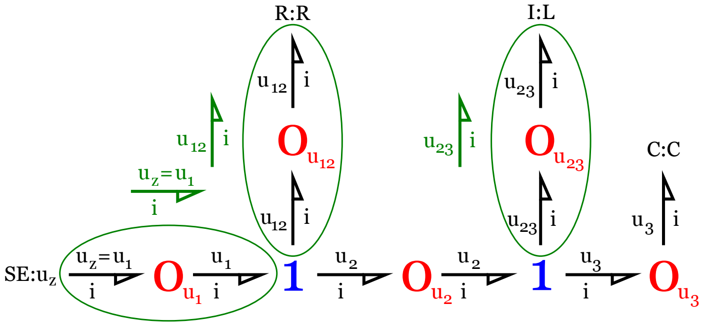


Fig. 5. Simplify the resulting graph by applying the simplification rules

Source: [6; 7].

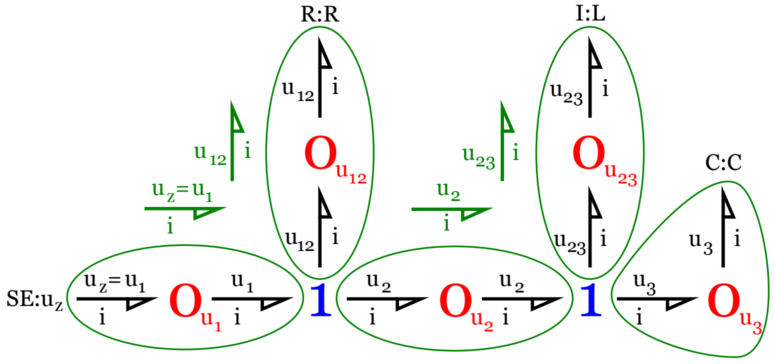


Fig. 6. Simplify the resulting graph by applying the simplification rules, the 0-junctions with voltages u_2 and u_3

Source: [6; 7].

A junction between two bonds can be left out, if the bonds have a through power direction (one bond incoming, the other outgoing). A bond between two the same junctions can be left out and the junctions can join into one junction. Two separately constructed identical effort or flow differences can join into one effort or flow difference. Determine the signal direction and causality in Fig. 7. Causality establishes the cause and effect relationships between the factors of power.

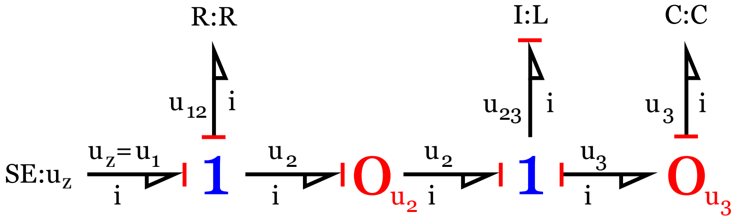


Fig. 7. Determine the causality

Source: [6; 7].

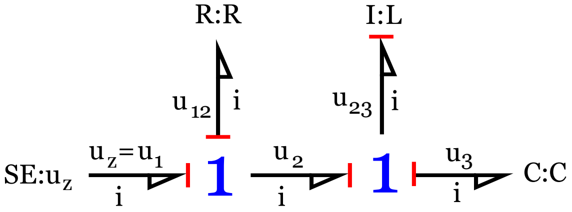


Fig. 8. Simplify the resulting graph, the 0-junctions reduction and causality

Source: [6; 7].

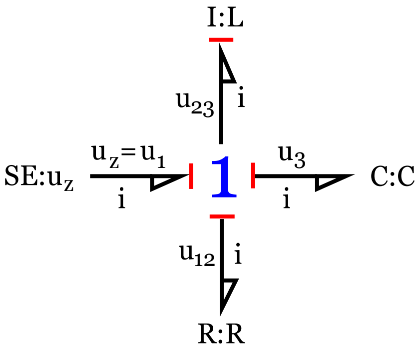


Fig. 9. The final bond graph with signal direction and causality

Source: [6; 7].

In bond graphs, the inputs and the outputs are characterized by the causal stroke. The causal stroke indicates the direction in which the effort signal is directed (by implication, the end of the bond that does not have a causal stroke is the end towards which the flow signal is directed). There are two ways of describing an element's behavior (e.g. effort in, flow out vs. flow in, effort out) as different causal forms. Note that the two alternative causal forms may, in general, require quite different mathematical operations. The causal form we use, i.e. which variable we select as input and which we select as output, can make a lot of difference. For example, the required mathematical operations may be well defined in one causal form, but not defined at all in the other. The causal bond graph of this system can be derived, in which the inputs and the outputs are characterized by the causal stroke. This is the starting point, from which we continue toward the differential equations describing the dynamics of the system. A causal bond graph (Fig. 9) can be expanded into a block diagram (Fig. 10).

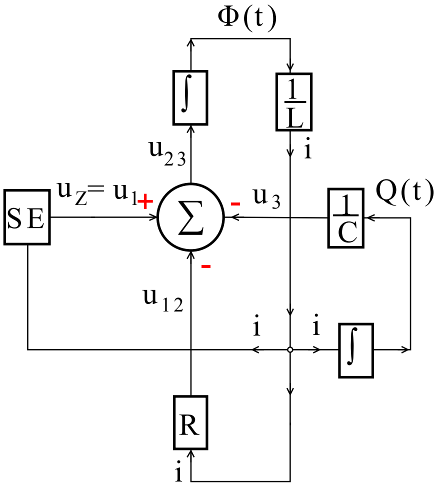


Fig. 10. Bond graph expanded to a block diagram with source effort voltage u_z

Source: [6; 7].

The equations derived from a bond graph diagram (Fig. 10) are:

$$\frac{d\,\Phi(t)}{dt} = u_{23} \tag{1}$$

$$\frac{dQ(t)}{dt} = i \tag{2}$$

$$u_{23} = -u_3 - u_{12} + u_Z \tag{3}$$

where state variables Φ(t) and Q(t) are magnetic flux and electric charge respectively and next equations:

$$u_3 = \frac{1}{C} \cdot Q(t) \tag{4}$$

$$u_{12} = R \,.\, i \tag{5}$$

$$i = \frac{1}{L} \,.\, \Phi(t) \tag{6}$$

State equation in matrix form are:

$$\begin{bmatrix} \dfrac{d\,\Phi(t)}{dt} \\ \dfrac{dQ(t)}{dt} \end{bmatrix} = \begin{bmatrix} -R.\dfrac{1}{L} & -\dfrac{1}{C} \\ \dfrac{1}{L} & 0 \end{bmatrix} . \begin{bmatrix} \Phi(t) \\ Q(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} . \, u_Z \tag{7}$$

After substitution:

$$\begin{bmatrix} \frac{di}{dt} \\ \frac{du_3}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} . \begin{bmatrix} i \\ u_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} .u_Z \qquad (8)$$

The state equation can be solved in Matlab/Simulink [13-18].

Conclusions. The differential equations describing the dynamics of the system in terms of the states of the system were derived from a bond graph diagram of a RLC circuit of electrical system. Model of a simple electrical RLC circuit consisting of a resistor, an inductor, and a capacitor is taken. The results correspond with equations obtained using traditional method, where the equations for individual components are created first and then the simulation scheme is derived on their basis, although the described method uses the reverse procedure. However, manual derivation of equations for larger systems is not all that simple. For instance, in some cases the derivation may lead to formation of so called algebraic loops. Similarly, complexities and errors of various types, like causal loops, power loops and differential causalities may exist in the model of a system.

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МЕТОДОЛОГІЯ ПОВ'ЯЗАНИХ ГРАФІВ У ЛАНЦЮГОВОМУ АНАЛІЗІ «RLC»

Актуальність теми дослідження. *Теорія зв'язних графів спрямована на вираження фізичних систем загального класу через енергетичні взаємодії. Факторами потужності є зусилля і потік. Вони мають різні тлумачення в різних фізичних областях. Тим не менш, потужність завжди може використовуватися як узагальнений ресурс для моделювання зв'язних систем, що знаходяться в декількох енергетичних областях.*

Постановка проблеми. *Формалізм силових графів дозволяє описати різні фізичні системи та їх взаємодії однаковим, алгоритмізованим способом та перетворити їх у опис простору станів. Це корисно для аналізу мехатронних систем, що перетворюють різні форми енергії (електричну, текучу, механічну) за допомогою інформаційних сигналів у механічну енергію, що отримується.*

Аналіз останніх досліджень і публікацій. *Протягом двох останніх десятиліть теорії графів приділялася увага університетів із усього світу, а зв'язні графи були частиною навчальних програм у все більшій кількості університетів. В останнє десятиліття їх промислове використання набуває все більшого значення. Метод Бондграфів був введений Генрі М. Пейнтером (1923-2002), професором MIT & UT Austin, який почав публікувати свої роботи з 1959 року і поступово напрацьовував термінологію та формальні методи, відомі сьогодні як зв'язні графи, що перекладаються як сполучені графи або графи ефективності.*

Виділення недосліджених частин загальної проблеми. *Модель електричної системи формується за допомогою вищезазначених формальних методів зв'язних графів. Поступово теорія силових графів у наведеному вище прикладі пояснюється аж до побудови рівнянь стану електричної системи. Потім рівняння стану розв'язуються за допомогою Matlab / Simulink.*

Виклад основного матеріалу. *Використання теорії зв'язних графів для моделювання електричної системи та перевірки її придатності для імітаційного моделювання електричних моделей. У різних варіаціях параметрів моделі ми можемо відстежувати її поведінку за різних умов експлуатації. Мова зв'язних графів прагне виразити фізичні системи загального класу через силові взаємодії. Фактори сили, тобто зусилля і потоку, мають різні інтерпретації в різних фізичних областях. Однак потужність завжди можна використовувати як узагальнену координату для моделювання зв'язаних систем, що знаходяться в декількох енергетичних областях.*

Висновки відповідно до статті. *Ми представили метод систематичної побудови графу зв'язків моделі електричної системи за допомогою Бондграфів. Практичний приклад електричної моделі наведено як приклад застосування цієї методології. Причинний аналіз також надає інформацію про правильність моделі. Диференціальні рівняння, що описують динаміку системи у термінах станів системи, були отримані із простого зв'язного графа електричної системи. Результати відповідають рівнянням, отриманим класичним ручним методом, де спочатку створюються рівняння для окремих компонентів, а потім на їх основі складається схема імітаційного моделювання. Запропонована методологія використовує зворотну процедуру. Однак вивести рівняння для більш складних систем вручну не так просто. Зв'язні графи виявляються придатним засобом для такого аналізу, серед електричних або інших систем.*

Ключові слова: *мехатроніка; енергетичне моделювання; Бондграфи; моделювання динамічної системи; джерело зусиль; джерело потоку; конденсатор; індуктор; резистор.*

Рис.: 4. Бібл.: 18.

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