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COMPUTER SIMULATION AND ANALYTICAL SOLUTION OF FOUR BAR MECHANISM

Urgency of the research. *The use of computers in technical practice leads to the extension of the possibility of solving mathematical models. This makes it possible to gradually automate complex calculations of equations of mathematical models. It is necessary to input the relevant inputs of the mathematical model, to build a simulation computer model and to monitor and evaluate the output results using a computer's output device.*

Target setting. *The possibilities of modeling a four-bar linkage mechanism by classical analytical methods and methods using computer modeling are presented in this paper. The problem is to describe the creation of a computer model and to show the mathematical model and its solution in the classical ways.*

Actual scientific researches and issues analysis. *The inspiration for the creation of the article was the study of the mechanisms in the work [1-3] and the study of other resources available in library and journal materials, as well as prepared study materials for students of Technical university Kosice.*

Uninvestigated parts of general matters defining. *The question of building a real mechanism model. The possibilities to building a real model, based on the result of simulation.*

The research objective. *The aim of this paper is to develop a functional model of the mechanism in ADAMS/View and Matlab and its complete kinematic analysis.*

The statement of basic materials.

The task was to create a computer model in MSC Adams and Matlab and to perform a four-bar linkage mechanism kinematic analysis. At the same time the classical procedure of analytical methods of kinematic analysis was described. Kinematic characteristics of driven members and their selected points were determined. The movement of the parts of the mechanism in its significant points was analyzed. The results of the solution were shown in both programs in graphical form. Kinematic analysis was performed by both vector and graphical methods. Finally, the results with a graphical representation of parameters such as angular displacement, angular velocity and angular acceleration of mechanism members are presented in this work. The results of these solutions are created in the form of graphs.

To ensure that the results do not differ from the model real, a good computer model gradually was created by its verification and modification, which is one of the advantages of MSC Adams.

The practical applicability of the mathematical model was limited by the existence of an analytical solution.

Conclusions. *The development of computer technology has expanded the limit of solvability of mathematical models and made it possible to gradually automate the calculation of equations of mathematical models. In a computer model the automated calculation can be treated as a real object sample. In various variations of calculation, we can monitor and measure the behavior of an object under different conditions, under the influence of different inputs. Graphical and vector methods were used for classical analytical methods. MSC Adams and Matlab were used for the automated calculations.*

Keywords: *kinematics, mechanical system, four bar linkage mechanism, computer simulation.*
Fig.: 13. References: 18.

Introduction. When solving the movements of machine parts, machines and equipment it is necessary to create a kinematic model first. The kinematic solution of the mechanisms is to detect individual driven members or some points of these members depending on the known or prescribed movement of the drive members. This involves determining the dependence of position, velocity and acceleration, rotation of angles, angular velocity, and angular acceleration of the investigated members and points depending on the movement of the drive member. The graphical and vector method also play an important role in the analytical solution of mechanisms. The kinematic model of the mechanism schematically captures all its properties, which are necessary in kinematic analysis, individual members with dimensions, kinematic pairs. The classical numerical solution of kinematics is often time consuming and difficult especially for complex kinematic models with different movements. Today, we can use a graphical solution to simplify and speed up the analysis. This is currently being replaced by a computer-based solution using a variety of software products. These software products facilitate the efforts of the solver. The solver enters the model configuration and input data, and the program then calculates the required outputs. These data can be presented in form of graphic results [1-4].

Description of the mechanical system. The presented paper shows the solution of kinematic analysis of mechanisms by classical methods of analytical mechanics and computer programs available today. Graphical method and vector method from analytical methods of solving

mechanisms and computer programs Matlab and MSC Adams are used. In the kinematic analysis of the model of four-bar mechanism, the following parts of the paper show the solution by the graphical method, then the model is created in the MSC Adams program and the vector method is shown and its solution is performed using Matlab.
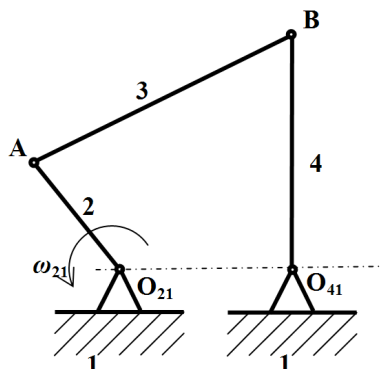

Fig. 1. Mechanical system of the four-bar mechanism

A four-bar mechanism shown in Fig.1 consists of four members, called bars or links. They are connected in a loop by four joints. A link connected to ground by a joint is called a crank. Driving link $O_{21}A$ has a counter clockwise angular velocity $\omega_{21}=1$ (rad/s). Our task is to determine angular displacement φ_{41}, angular velocity ω_{41} and angular acceleration α_{41} of the link $O_{41}B$ graphically for the crank position indicated and then to create a model of a mechanism in MSC Adams/View and next in MATLAB environment. The crank $O_{21}A$ rotates around point O_{21}, motion of the connecting member AB is a general plane motion and member $O_{41}B$ rotates around point O_{41}. We indicate velocities of the points A and B [1-5].

Examples of mechanical systems. In engineering practice we can meet the following forms of the four-bar mechanism of mechanical systems. Examples of mechanisms are shown in Fig. 2 and Fig. 3 [1].
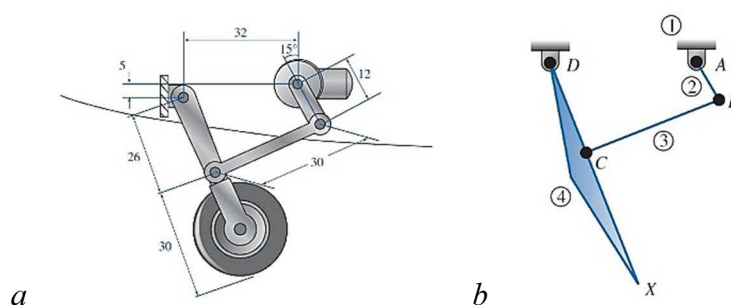

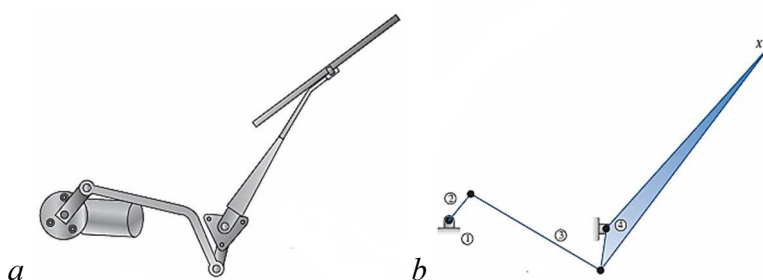*Fig. 2. Mechanical system: a – nose wheel assembly; b – kinematic scheme [1]*

Fig. 3. Mechanical system: a – rear-window wiper mechanism; b –kinematic scheme [1]

Kinematics analysis of the model. The velocity of points A and B can be obtained by using the rule of the viewing angles [4]. Tangent of the angle β_{v21} under which we see the endpoints

of velocity from the permanent center of rotation is proportional to the angular velocity of the rotating member (Fig. 4).
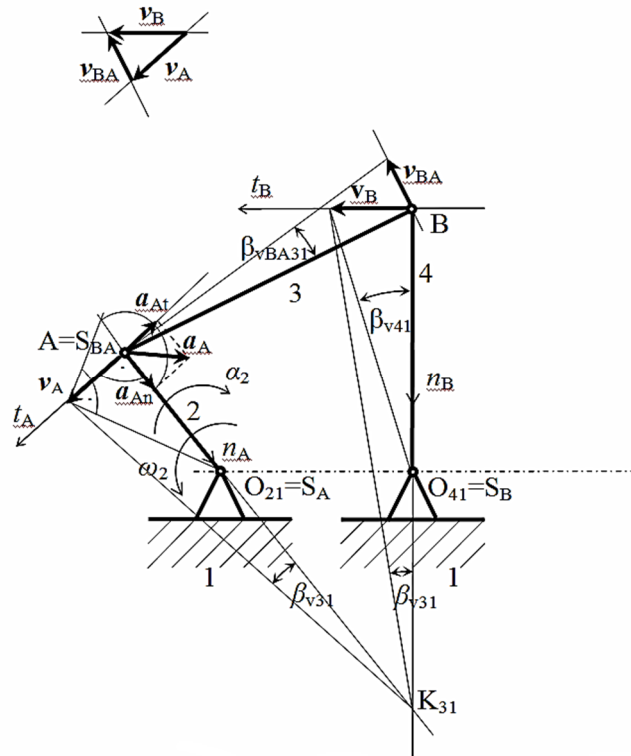

Fig. 4. The four-bar mechanism with velocity in point A and velocity in point B

If we denote the angular velocity ω_{21} of the member 2 with respect to 1, then for the velocity v_A and acceleration a_{An} at point A shall apply [5-8]:

$$v_A = \overline{O_{21}A} \cdot \omega_{21} \qquad (1)$$

Normal component of the acceleration in point A:

$$a_{An} = \frac{v_A^2}{O_{21}A} \qquad (2)$$

Angular velocity ω_{21}:

$$tg\beta_{v21} = \frac{v_A}{O_{21}A} = \dot{\varphi}_{21} = \omega_{21} \qquad (3)$$

Angular velocity ω_{31}:

$$tg\beta_{v31} = \frac{v_A}{K_{31}A} = \frac{v_B}{K_{31}B} = \omega_{31} \qquad (4)$$

Velocity of point B:

$$v_B = \overline{K_{31}B} \cdot \frac{v_A}{K_{31}A} \qquad (5)$$

Angular velocity ω_{41} of the member 4 we determine:

$$\omega_{41} = \frac{v_B}{O_{41}B} \qquad (6)$$

The member 3 is moving in the plane in form 3:1=3:2+2:1 and also 3:1=3:4+4:1. For the velocity of the point A:

$$v_{A31} = v_{A32} + v_{A21} \tag{7}$$

For the velocity of the point B:

$$v_{B31} = v_{B34} + v_{B41} \tag{8}$$

$$v_{B31} = v_{A31} + v_{BA31} \tag{9}$$

Acceleration a_B of the point B [6-7]:

$$a_B = a_A + a_{BA} \tag{10}$$

Where a_{Bt} – known direction,

a_{Bn} – can be obtained by Euclidean construction,

a_{At} – is zero because

$$\omega_{21} = konst \Rightarrow \alpha_{21} = 0, a_{At} = \alpha_{21} \cdot \overline{O_{21}A} = 0 \tag{11}$$

a_{An} – is normal component of the acceleration of the point A:

$$a_{An} = \frac{v_A^2}{\overline{O_{21}A}} \tag{12}$$

Graphical solution of the above vector equation is in the Fig. 4. Angular velocity ω_{41} and angular acceleration α_{41} is then calculated according to:

$$\omega_{41} = \frac{v_B}{\overline{O_{41}B}} \tag{13}$$

$$\alpha_{41} = \frac{a_{Bt}}{\overline{O_{41}B}} \tag{14}$$

In Fig. 4 there is the model in graphic form with velocities of points A and B.
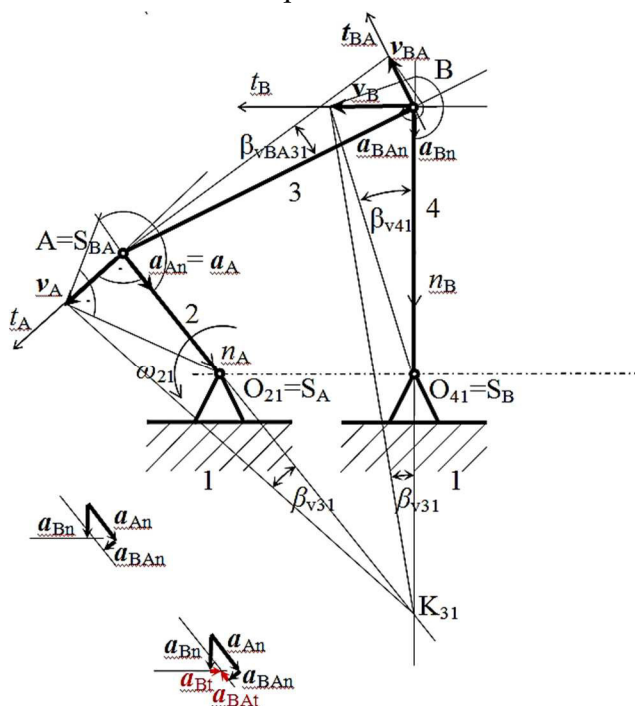Normal component of the acceleration in point A and B is shown in Fig. 5.

Fig. 5. The four-bar mechanism with acceleration in point A and velocity in point B

Model of the four-bar mechanism using MSC Adams/View. The given four-bar mechanism was modeled in MSC ADAMS/View and the initial parameters were provided [1-5]. In the initial window of the program MSC Adams we set data folder, name of the project, units and the working grid. We created the individual bodies of the mechanism. We selected the rigid body link from the Toolbox.
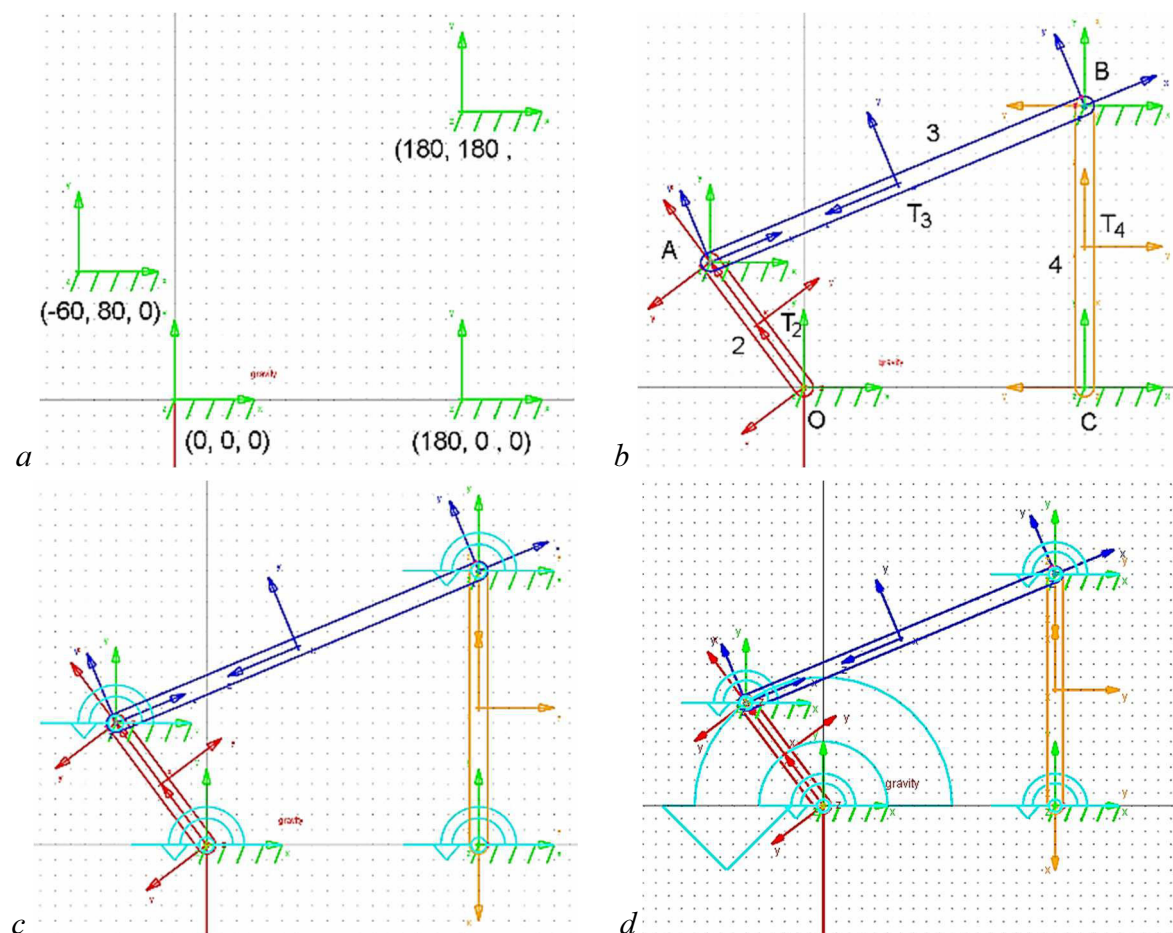

Fig. 6. The four-bar mechanism and creation:
a – points; b – members; c – joints; d – motion

We defined the geometry, length, width and depth. Driving link $O_{21}A$ (Fig. 1) has a counterclockwise angular velocity $\omega_{21}=1$ rad/s, [8-12]. We select point tool in Main Toolbox. Move the cursor to (0, 0, 0) position and points 1, 2, 3 and 4 are created a named automatically (Fig.6, a). Then move the cursor to (180, 0, 0) position and created the member 4 (Fig.6, b). The member 2 should be rotated about point from horizontal position to counter clockwise rotate. Then connected the members 3 with member 2 and 4 (Fig. 6, b). Next we created joints between the members and ground (Fig. 6, c). Select the rotational joint motion tool with speed 1 rad/s [6-7] (Fig. 6, d).

The final model of the mechanism is shown in Fig. 7. We create the measure windows for angle φ_{41}, angular velocity ω_{41} and angular acceleration α_{41} of the member 4. Then we start simulation and we determine angle displacement φ_{41}, angular velocity ω_{41} and angular acceleration α_{41} of the member 4 (Fig. 8-11). The graph in Fig. 8a-b) describes the motion of the member AB.
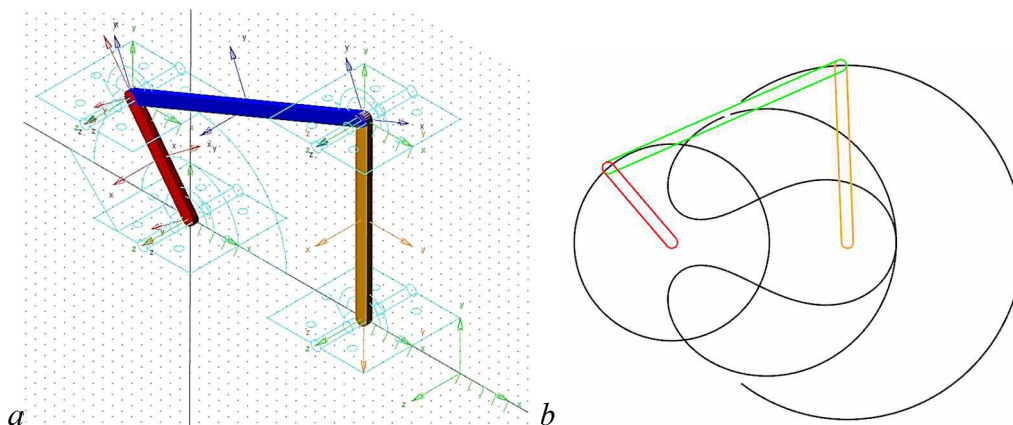
Fig. 7. Model of a four-bar mechanism in MSC Adams View (a), trajectory of the points A, B and x_{T3} of the members of mechanism (b)
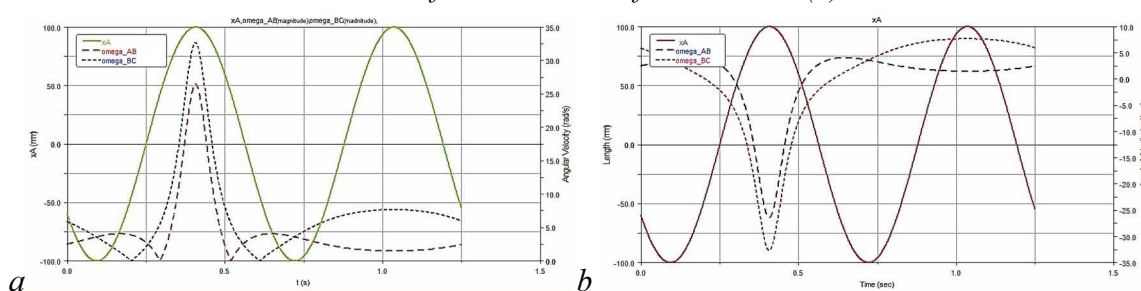

Fig. 8. Displacement $x_A = f(t)$, magnitude of angular velocity $\omega_{AB} = f(t)$ and $\omega_{BC} = f(t)$ (a); displacement $x_A = f(t)$, angular velocity $\omega_{AB} = f(t)$ and $\omega_{BC} = f(t)$ (b)
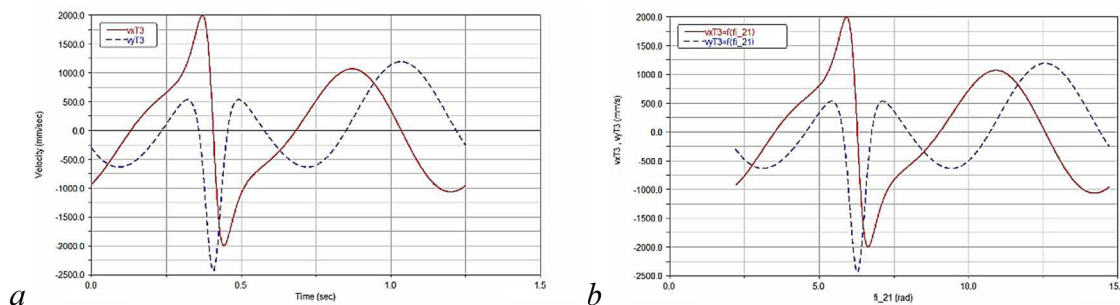

Fig. 9. Velocity $v_{xT3} = f(t)$ and $v_{yT3} = f(t)$, b) velocity $v_{xT3} = f(\varphi_{21})$ and $v_{yT3} = f(\varphi_{21})$
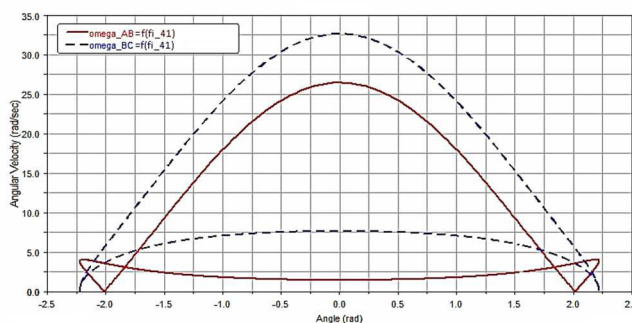

Fig. 10. The angular velocity $\omega_{AB} = f(\varphi_{41})$ and $\omega_{BC} = f(\varphi_{41})$

In the window of the Postprocessor we selected window with animation of the mechanism and windows with results data in graphics form for angle φ_{41}, angular velocity ω_{41} and angular acceleration α_{41} of the member 4 (Fig. 11).
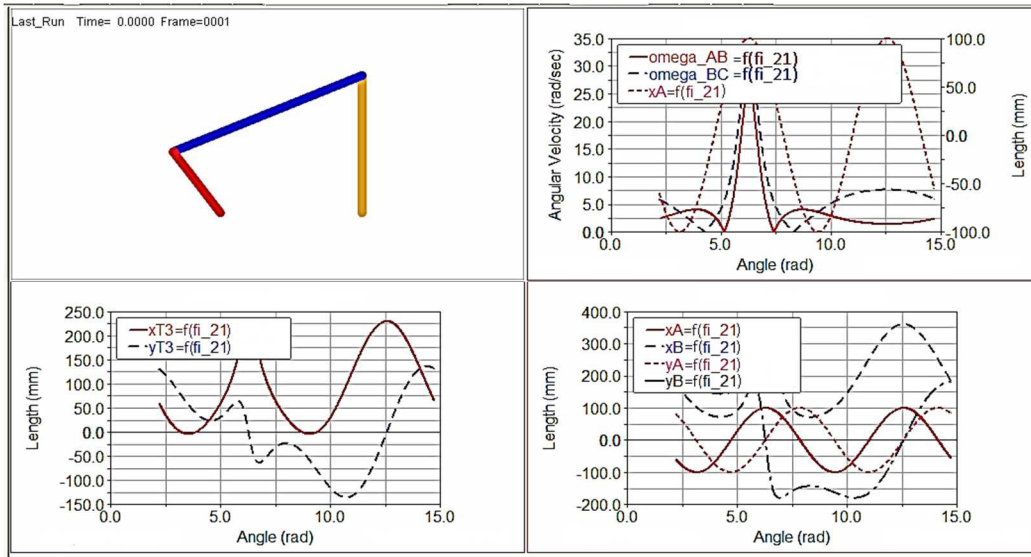
Fig. 11. Illustration of the Postprocessor windows with model of mechanism, angular velocity ω_{AB}, angular velocity ω_{BC}, displacement of x_A, x_B, y_A, y_B, x_{T3}, y_{T3} versus angle φ_{21}

Simulation of four-bar mechanism in Matlab. When applying the vector method, we replace the mechanism with vectors that connect individual kinematic pairs. The result is a closed polygon in Fig.12.
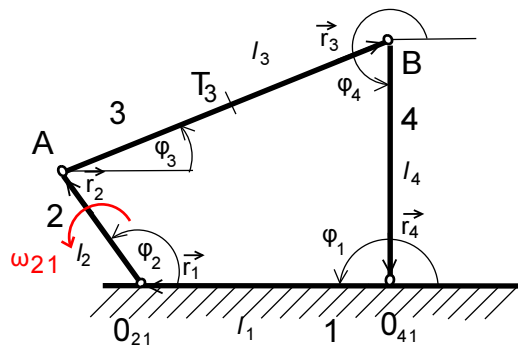

Fig. 12. Indication of vectors in vector method

We write a vector equation (15) for this polygon (Fig.12) and two scalar equations (16) and (17) for its components. The matrix equation of the polygon [8-13]:

$$\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4 = \mathbf{0}. \tag{15}$$

Scalar equations in form:

$$l_1\cos\varphi_1 + l_2\cos\varphi_2 + l_3\cos\varphi_3 + l_4\cos\varphi_4 = 0 \tag{16}$$

$$l_1\sin\varphi_1 + l_2\sin\varphi_2 + l_3\sin\varphi_3 + l_4\sin\varphi_4 = 0 \tag{17}$$

After the differentiation of the equations (16) and (17) over time, we obtain the equations for determining angular velocities shown in matrix form:

$$\begin{bmatrix} l_3\sin(\varphi_3) & l_4\sin(\varphi_4) \\ l_3\cos(\varphi_3) & l_4\cos(\varphi_4) \end{bmatrix} \cdot \begin{bmatrix} \dot{\varphi}_3 \\ \dot{\varphi}_4 \end{bmatrix} = \begin{bmatrix} -l_2\dot{\varphi}_2\sin(\varphi_2) \\ -l_2\dot{\varphi}_2\cos(\varphi_2) \end{bmatrix} \tag{18}$$

After the differentiation of the equations (18) over time, we obtain the equations for determining angular acceleration in matrix form:

$$\begin{bmatrix} l_3\sin(\varphi_3) & l_4\sin(\varphi_4) \\ l_3\cos(\varphi_3) & l_4\cos(\varphi_4) \end{bmatrix} \cdot \begin{bmatrix} \ddot{\varphi}_3 \\ \ddot{\varphi}_4 \end{bmatrix} == \begin{bmatrix} -l_2\ddot{\varphi}_2\sin(\varphi_2) - l_2\dot{\varphi}_2^2\cos(\varphi_2) - l_3\dot{\varphi}_3^2\cos(\varphi_3) - l_4\dot{\varphi}_4^2\cos(\varphi_4) \\ -l_2\ddot{\varphi}_2\cos(\varphi_2) + l_2\dot{\varphi}_2^2\sin(\varphi_2) + l_3\dot{\varphi}_3^2\sin(\varphi_3) + l_4\dot{\varphi}_4^2\sin(\varphi_4) \end{bmatrix} \tag{19}$$

These problems can also be solved for example, using the Matlab [13-18]. The results of the simulation are in (Fig. 13a-b), where:

$$\dot{\varphi}_{31} = \omega_{31}, \quad \dot{\varphi}_{41} = \omega_{41}, \quad \ddot{\varphi}_{31} = \alpha_{31}, \quad \ddot{\varphi}_{41} = \alpha_{41}.$$
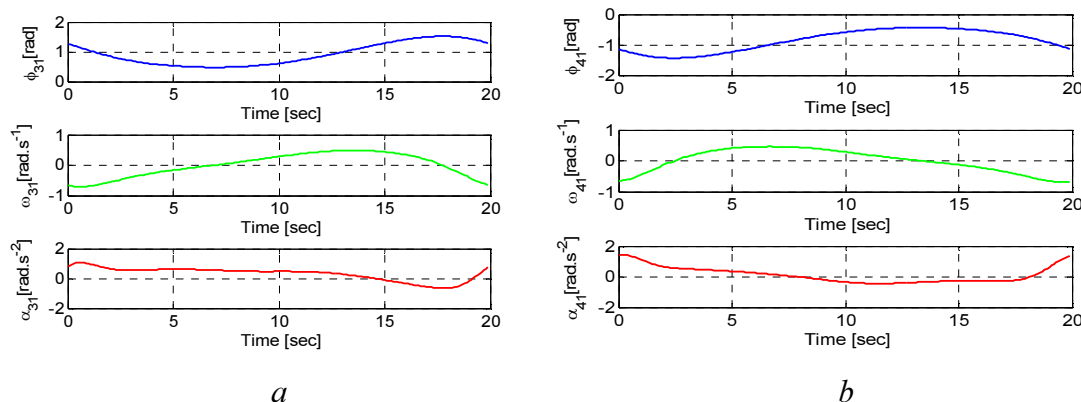

a   *b*

Fig. 13. The angle-time, angular velocity-time and angular acceleration-time graph of the member 3 (a), of the member 4 (b)

Conclusions. The paper presents the possibilities of modeling a four-bar linkage mechanism by classical analytical methods and computer methods using computer modeling. The aim of this paper is to develop a functional model of the mechanism in ADAMS/View and its complete kinematic analysis. Furthermore a solution is also realized in the Matlab software. We analyze the movement of the members of the mechanism of its significant points. Kinematic analysis was performed with vector and graphical methods. Finally, the work presents results with a graphical representation of parameters such as angular displacement, angular velocity and angular acceleration of mechanism members. The results of these solutions are created in the form of graphs.

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КОМП'ЮТЕРНЕ МОДЕЛЮВАННЯ ТА АНАЛІТИЧНЕ РІШЕННЯ ЧОТИРИЛАНКОВОГО МЕХАНІЗМУ

Актуальність теми дослідження. *Використання комп'ютерів в технічній практиці призводить до розширення можливостей створення математичних моделей. Це дає можливість поступово автоматизувати складні обчислення рівнянь математичних моделей. Необхідно ввести відповідні вхідні дані математичної моделі, побудувати імітаційну комп'ютерну модель, а також контролювати і оцінювати вихідні результати за допомогою пристрою комп'ютерної периферії.*

Постановка проблеми. *У статті представлені можливості моделювання чотириланкового важільного механізму класичними аналітичними методами і методами комп'ютерного моделювання. Завдання полягає в тому, щоб описати створення комп'ютерної моделі і показати математичну модель та її розв'язання класичними способами.*

Аналіз останніх досліджень і публікацій. *Натхненням для створення статті послужило вивчення механізмів в роботах [1-3] і вивчення інших ресурсів, доступних в бібліотечних та журнальних матеріалах, а також підготовлені навчальні матеріали для студентів Технічного університету Кошице.*

Виділення недослідженних частин загальної проблеми. *Питання побудови реальної моделі механізму. Можливості побудови реальної моделі на основі результату моделювання.*

Мета статті. *Метою даної роботи є розробка функціональної моделі механізму в ADAMS / View і Matlab та його повний кінематичний аналіз.*

Виклад основного матеріалу. *Завданням було створити комп'ютерну модель в MSC Adams і Matlab та виконати кінематичний аналіз чотириланкового важільного механізму. Одночасно була описана класична процедура аналітичних методів кінематичного аналізу. Були визначені кінематичні характеристики керованих членів та вибрані ними точки. Проаналізовано рух частин механізму у його значущих точках. Результати рішення були показані в обох програмах у графічному вигляді. Кінематичний аналіз проводили як векторними, так і графічними методами. Нарешті, в цій роботі представлені результати з графічним зображенням таких параметрів, як кутове переміщення, кутова швидкість та кутове прискорення елементів механізму. Результати цих рішень представлені у вигляді графіків. Щоб результати не відрізнялися від реальних моделей, поступово була створена хороша комп'ютерна модель шляхом її перевірки та модифікації, що є однією з переваг MSC Adams. Практична придатність математичної моделі була обмежена існуванням аналітичного рішення.*

Висновки відповідно до статті. *Розвиток обчислювальної техніки розширив межу розв'язності математичних моделей і дозволив поступово автоматизувати обчислення рівнянь математичних моделей. У комп'ютерній моделі автоматизований розрахунок можна розглядати як реальний зразок об'єкта. У різних варіантах розрахунків ми можемо контролювати та вимірювати поведінку об'єкта за різних умов, під впливом різних вхідних даних. Для класичних аналітичних методів використовувались графічний та векторний методи. Для автоматизованих розрахунків використовували MSC Adams і Matlab.*

Ключові слова: *кінематика; механічна система; чотириланковий механізм зчеплення; комп'ютерне моделювання.*

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