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SMALL SIGNAL MODEL OF SINGLE-PHASE QUASI-Z-SOURCE INVERTER WITH COUPLED INDUCTORS

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МАЛОСИГНАЛЬНА МОДЕЛЬ ОДНОФАЗНОГО КВАЗІ-ІМПЕДАНСНОГО ІНВЕРТОРА З МАГНІТО-ЗВ'ЯЗАНИМИ ІНДУКТИВНОСТЯМИ

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МАЛОСИГНАЛЬНАЯ МОДЕЛЬ ОДНОФАЗНОГО КВАЗИ-ИМПЕДАНСНОГО ИНВЕРТОРА С МАГНИТО-СВЯЗАННЫМИ ИНДУКТИВНОСТЯМИ

Focus is on the single-phase quasi-Z-source inverter with coupled inductor. State space averaging method is used for small-signal analysis. Small signal behavior of the converter expressed analytically in form of set of transfer functions. It can be used for controller design in closed loop system and analysis of inductor coupling.

Key words: single-phase inverter, qZS network, small signal model.

Розглянуто однофазний квазі-імпедансний інвертор з магніто-зв'язаними індуктивностями. Проведено лінеаризацію за методом усереднення в просторі станів. Складено матричний вираз, що описує в p-області поведінку перетворювача для малого сигналу. Аналітично отримано передавальні функції, які можна використовувати для синтезу замкнутої системи керування та проводити аналіз впливу магнітного зв'язку між індуктивностями.

Ключові слова: однофазний інвертор, квазі-імпедансна ланка, малосигнальна модель.

Рассмотрен однофазный квази-импедансный инвертор с магнито-связанными индуктивностями. Проведена линеаризация по методу усреднения в пространстве состояний. Составлено матричное выражение, описывающее в p-области поведение преобразователя для малого сигнала. Аналитически получены передаточные функции, которые можно использовать для синтеза замкнутой системы и анализа влияния магнитных связей между индуктивностями.

Ключевые слова: однофазный инвертор, квази-импедансное звено, мало-сигнальная модель.

Introduction. Renewable energy capacity is growing rapidly. These energy sources supplied 21,7 % of the world electricity consumption in 2014 [1]. In particular, solar Photovoltaic (PV) and wind power grew by almost 45,2 % and 22,2 % during the last years respectively [1].

Dc-ac converters are used to inject renewable energy into the grid. Traditionally, Voltage Source Inverters (VSIs) or Current Source Inverters (CSIs) are used. However, typically they cannot provide more than double input voltage regulation ratio because of the losses. To overcome that drawback, intermediate voltage boost dc-dc converters are used. At the same time, this solution as well as the control system are topologically complex due to the two-stage power conversion.

Z-source inverters overcome the limitation of the conventional inverters: they have buck, boost operation capability and do not suffer from short circuit. The Quasi-Z-Source Inverter e (qZSI) has appeared as a derivation from the Z-source inverter. This circuit has continuous input current and used in different application [2–5]. Many review and comparison papers have been published [6; 7].

At the same time, the qZSI with coupled inductors is not studied enough. In particular influence of coupling on the input current ripple and dynamic behaviour.

Objective of the paper. The main goal of the paper is a composition of the small signal model of the qZSI with coupled inductor. The small signal model will allow further analysis of the current and voltage ripple in the passive components. Also dynamic behavior analysis can be done by means of small signal model. Fig. 1 illustrates investigated topology of the single-phase qZSI.
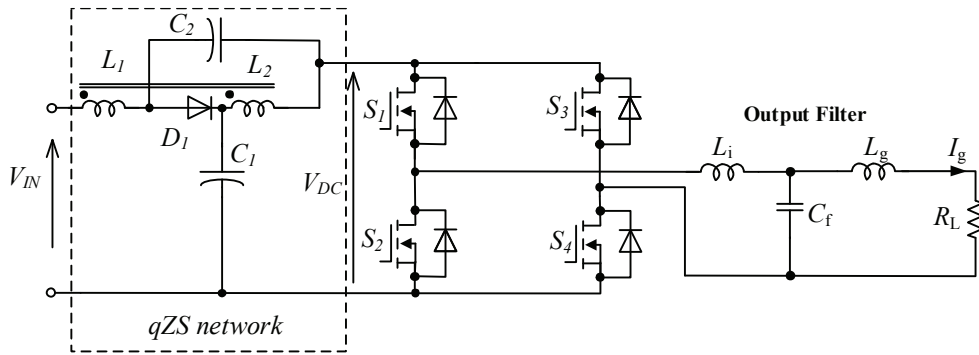
Fig. 1. Investigated topology of the single-phase qZSI

Main part of the paper. The averaging method in a state space is used to analyze the stability of PWM converters [8] and current and voltage fluctuating [9]. In general, switching period of the any switching converter is broken into intervals on which it is replaced by a linear circuit. On the i-th interval of his work describes the system of equations:

$$\begin{cases} K\dfrac{dx(t)}{dt} = A_i x(t) + B_i u(t), \\ y(t) = C_i x(t) + E_i u(t), \end{cases} \tag{1}$$

where K, A_i, B_i, C_i, E_i – the matrix of coefficients on the i-th interval; $x(t)$ – the state vector; $u(t)$ – vector of input variables; $y(t)$ – vector of output variables. Hereinafter, bold matrices and vectors. As states decided to choose a minimum set of voltages and currents in containers inductances, which completely describe the system.

Further averaging system of equations over the period of the converter. Averaged model of the system of equations in matrix form:

$$\begin{cases} \dfrac{d\langle \mathbf{x}(t)\rangle_T}{dt} = \mathbf{A}_{av}\langle \mathbf{x}(t)\rangle_T + \mathbf{B}_{av}\langle \mathbf{u}(t)\rangle_T, \\ \langle \mathbf{y}(t)\rangle_T = \mathbf{C}_{av}\langle \mathbf{x}(t)\rangle_T + \mathbf{E}_{av}\langle \mathbf{u}(t)\rangle_T. \end{cases} \tag{2}$$

where A_{av}, B_{av}, C_{av} and E_{av} – averaged coefficient matrix describing the behavior of the adjustable in the averaged values. Here and below, angle brackets $\langle \rangle_T$ denote averaging of magnitude over the period of switching T. We write more, which are averaged matrix.

Further, all state averages, input voltage and the control signal represented as a constant component and a variable component of small amplitude with a frequency below the switching frequency drive:

$$\begin{cases} \langle x(t)\rangle_T = X + \tilde{x}(t); \langle u(t)\rangle_T = U + \tilde{u}(t)\langle y(t)\rangle_T = Y + \tilde{y}(t), \\ \langle d_i(t)\rangle_T = D_i + \tilde{d}(t). \end{cases} \tag{3}$$

In order to estimate values of the passive components and dynamic properties the steady state analysis is detailed below. The operating period of the qZSI in the CCM can be divided into only two time intervals and can be represented by means of two equivalent circuits. It means that all switching states can be separated into two modes: active states (Fig. 2, *a*) and ST state (Fig. 2, *b*) correspondently.

The magnetically coupled inductors are represented by means of an equivalent circuit that contains an ideal transformer with a turns ratio $N_1{:}N_2$, magnetizing inductance L_M and leakage inductance L_L [10; 11].
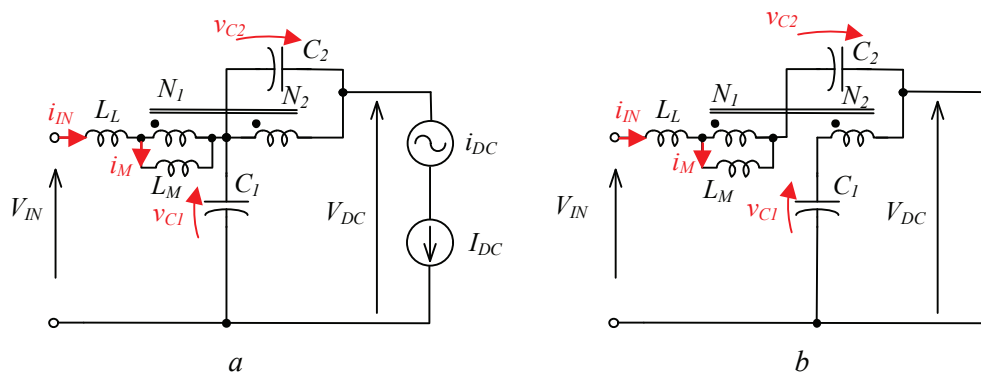
Fig. 2. Equivalent circuits of the proposed topology: a – active states; b – shoot-through states

Output side of the inverter is represented by means of the dc and ac current sources. It models power flow from the input to the grid or load through the dc-link.

It has constant and variable components, which is the characteristic of the single-phase system:

$$i_{DC} = I_{DC} + \tilde{i}_{DC}, \qquad \tilde{i}_{DC} = I_{MAX} \quad sin(m\omega t), \qquad (4)$$

where I_{DC} is a constant component of the load current, $\tilde{i}_{DC}$ is a variable component, I_{MAX}. is the peak value of the variable component. Both I_{DC} and $\tilde{i}_{DC}$ will have influence on the state variables of the system, including i_{IN}, i_M, v_{C1}, and v_{C2}, all of which also consist of two parts: the dc component and the $m\omega$ component:

$$i_{IN} = I_{IN} + \tilde{i}_{IN}, i_M = I_M + \tilde{i}_M, v_{C1} = V_{C1} + \tilde{v}_{C1}, v_{C2} = V_{C2} + \tilde{v}_{C2}. \qquad (5)$$

Finally, time intervals are defined and denoted as: T_S – ST state time period; T_a – active state time period. In terms of dc components, the variable components are omitted. The active state time interval can be described by following differential equations:

$$\begin{cases} L_L \dfrac{di_{IN}}{dt} = V_{IN} - v_{C1} + \dfrac{N_1}{N_2} v_{C2}, \\[2mm] L_M \dfrac{di_M}{dt} = -\dfrac{N_1}{N_2} v_{C2}, \\[2mm] C_1 \dfrac{dv_{C1}}{dt} = i_{IN} - I_{DC}, \\[2mm] C_2 \dfrac{dv_{C2}}{dt} = -(i_{IN} - i_M)\dfrac{N_1}{N_2} - I_{DC}. \end{cases} \qquad (6)$$

In its turn ST state can be described:

$$\begin{cases} L_L \dfrac{di_{IN}}{dt} = V_{IN} + v_{C2}\dfrac{N_1}{N_2} - v_{C1}, \\[2mm] L_M \dfrac{di_M}{dt} = \dfrac{N_1}{N_2} v_{C1}, \\[2mm] C_1 \dfrac{dv_{C1}}{dt} = (i_{IN} - i_M)\dfrac{N_1}{N_2}, \\[2mm] C_2 \dfrac{dv_{C2}}{dt} = -i_{IN}. \end{cases} \qquad (7)$$

After that, it can be derived the averaged equations that can be represented in matrix form like in equation (2).

$$\begin{cases} L_L \dfrac{d<i_{IN}>}{dt} = (1-D)\ (V_{IN} - v_{C1} + \dfrac{N_1}{N_2} v_{C2}) + D\cdot(V_{IN} + v_{C2} - \dfrac{N_1}{N_2} v_{C1}) = 0, \\[2mm] L_M \dfrac{d<i_M>}{dt} = (1-D)\cdot\dfrac{N_1}{N_2} v_{C2} + D\dfrac{N_1}{N_2} v_{C1} = 0, \\[2mm] C_1 \dfrac{d<v_{C1}>}{dt} = (1-D)\cdot(i_{IN} - I_{DC}) + D(i_{IN} i_M)\dfrac{N_1}{N_2} = 0, \\[2mm] C_2 \dfrac{d<v_{C2}>}{dt} = (1-D)\cdot(-(i_{IN} - i_M)\cdot\dfrac{N_1}{N_2} - I_{DC} - D\cdot i_{IN} = 0, \end{cases} \quad (8)$$

where the D – is ST duty cycle.

In terms of variable components, the input dc power supply keeps the voltage constant and it does not contain the $m\omega$ component.

Let us consider dynamic equations during the active state output voltage generation. The dynamic equations for active state are:

$$\begin{cases} L_L \dfrac{d\tilde{i}_{IN}}{dt} = \dfrac{N_1}{N_2} \tilde{v}_{C2} - \tilde{v}_{C1}, \\[2mm] L_L \dfrac{d\tilde{i}_M}{dt} = -\dfrac{N_1}{N_2} \tilde{v}_{C2}, \\[2mm] C_1 \dfrac{d\tilde{v}_{C1}}{dt} = \tilde{i}_{IN} - i_{DC}, \\[2mm] C_2 \dfrac{d\tilde{v}_{C2}}{dt} = -(\tilde{i}_{IN} - \tilde{i}_M)\dfrac{N_1}{N_2} - i_{DC}. \end{cases} \quad (9)$$

Finally, during the ST state operation, it can be written as:

$$\begin{cases} L_L \dfrac{d\tilde{i}_{IN}}{dt} = \tilde{v}_{C2} - \dfrac{N_1}{N_2} \tilde{v}_{C1}, \\[2mm] L_L \dfrac{d\tilde{i}_M}{dt} = \dfrac{N_1}{N_2} \tilde{v}_{C1}, \\[2mm] C_1 \dfrac{d\tilde{v}_{C1}}{dt} = (\tilde{i}_{IN} - \tilde{i}_M)\dfrac{N_1}{N_2}, \\[2mm] C_2 \dfrac{d\tilde{v}_{C2}}{dt} = -\tilde{i}_{IN}. \end{cases} \quad (10)$$

As $\tilde{x}$ is an $m\omega$ component, the average value of the variables $\tilde{x}$ over one switch cycle T is not zero, as a result, it can be expressed as:

$$
\begin{cases}
L_L \dfrac{d<\tilde{i}_{IN}>}{dt} = (1-D)\cdot(\dfrac{N_1}{N_2}\tilde{v}_{C2} - \tilde{v}_{C1}) + D\cdot(\tilde{v}_{C2} - \dfrac{N_1}{N_2}\tilde{v}_{C1}), \\[2mm]
L_L \dfrac{d<\tilde{i}_M>}{dt} = (1-D)\cdot\dfrac{N_1}{N_2}\tilde{v}_{C2} + D\cdot(\dfrac{N_1}{N_2}\tilde{v}_{C1}), \\[2mm]
C_1 \dfrac{d<\tilde{v}_{C1}>}{dt} = (1-D)\cdot(\tilde{i}_{IN} - i_{DC}) + D\cdot(\tilde{i}_{IN} - \tilde{i}_M)\cdot\dfrac{N_1}{N_2}, \\[2mm]
C_2 \dfrac{d<\tilde{v}_{C2}>}{dt} = (1-D)\cdot(-(\tilde{i}_{IN} - \tilde{i}_M)\cdot\dfrac{N_1}{N_2} - \tilde{i}_{DC}) - D\cdot\tilde{i}_{IN}.
\end{cases}
\tag{11}
$$

Using Eqs. (11), dependences i_{IN}, i_M, v_{C1}, $v_{C2} = f(I_{MAX}, D, C, m, \omega, L_M, L_L)$ can be obtained for further low frequency ripple and dynamic analysis.

Conclusions. This paper has presented small signal model of the single-phase quasi-Z-source inverter with coupled inductors. State space averaging method is used for small-signal analysis. Small signal model can be used for further stability analysis, controller design in closed loop system, passive components selection and analysis of inductor coupling.

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