DOI: https://doi.org/10.25140/2411-5363-2026-2(44)-131-137
UDC 539.3

Serhii Trubachev[1], Valerii Kolodezhnyi[2]

[1]PhD in Technical Sciences, Docent,
Associate Professor of the Dynamics and Strength of Machines and Strength of Materials Department
National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute» (Kyiv, Ukraine)
E-mail: strubachev1@ukr.net. **ORCID**: https://orcid.org/0000-0002-7349-9426
ResearcherID: AAK-6228-2020. **Scopus Author ID**: 16437921100

[2]Senior Lecturer of the Dynamics and Strength of Machines and Strength of Materials Department
National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute» (Kyiv, Ukraine)
E-mail: bokshin@ukr.net. **ORCID:** https://orcid.org/0000-0002-1722-1390. **ResearcherID:** l-CtlTMAAAAJ

DETERMINATION OF DYNAMIC CHARACTERISTICS OF RODS OF SHIPBUILDING STRUCTURES

Modern shipbuilding is developing towards increasing the tonnage of ships and the power of their power plants, which is accompanied by the active use of lightweight structural materials. In such conditions, the problem of ensuring the vibration strength and reliability of hull structures becomes critically acute, since uncontrolled vibrations of core elements - the basic components of the hull set, masts and shaft lines - lead to fatigue failure, a decrease in the accuracy of navigation equipment and a deterioration in conditions for the crew.

Traditional analytical methods have limited application for real objects with complex geometry and variable stiffness, and numerical methods require effective algorithms for accurate prediction of dynamic characteristics at the design stage.

In the work, a comprehensive methodology for determining the natural forms and frequencies of oscillations of core elements of both constant and variable cross-section using a variational-grid approach was developed and scientifically substantiated.

The iterative method of coordinate descent has also been further developed, the application of which in rod dynamics problems allows you to avoid computational difficulties associated with the formation and operation with global mass and stiffness matrices. In addition, the application of the coordinate descent method in vibration problems allows you to solve high-dimensional problems using only the RAM of the PC. Rounding errors do not accumulate.

Analytical expressions were obtained to determine the main natural frequencies of longitudinal and bending vibrations for rods, the physical and geometric characteristics of which vary according to binomial laws. The nature of the influence of the parameters of taper and wedge-shape on the spectrum of natural frequencies was studied. The developed approach was tested on examples of the calculation of cantilever rods, which allows you to design ship structures with predetermined dynamic properties to avoid resonance phenomena.

Keywords: *oscillations; dynamic characteristics; rod elements; variable cross-section; variational-grid approach; coordinate descent method.*

References: 7.

Relevance of the research. The current stage of development of world shipbuilding is characterized by a tendency to increase the tonnage of ships, increase the power of the main power plants and the widespread introduction of lightweight structural materials. In these conditions, the problem of ensuring the vibration strength and reliability of ship structures becomes critical. Rod elements are the basic components of the hull assembly, foundations of mechanisms, masts and shaft lines. Uncontrolled vibrations of these elements lead to premature fatigue failure, a decrease in the accuracy of navigational instruments and a deterioration in the conditions for the crew on board.

Problem statement. In modern engineering practice, the basic approaches for assessing the condition of rods and complex systems based on them are the approaches of material resistance [1] and structural mechanics [2]. Since such structures are constantly under the influence of dynamic factors during use, the study of vibration processes in rods with a constant or variable cross-sectional area is of particular importance. The variability of fixation methods (boundary conditions) necessitates the use of mathematical modeling methods for vibration analysis. At the same time, if the results can be described using functional series, priority, is usually, given to analytical methods. Of particular scientific interest is how geometric parameters (for example, narrowing or wedge-shape) correlate with the frequency characteristics of longitudinal and transverse vibrations.

The key difficulty in designing for vibration resistance is to find the spectrum of natural frequencies and the corresponding configurations (shapes) of vibrations. Mathematically, this procedure is usually, implemented by solving the classical generalized eigenvalue problem:

$$(Ku,v)=\omega^2(Mu,v),\ \forall v\in V, \tag{1}$$

where V – set of admissible functions, (Ku,v), (Mu,v) – a family of symmetric bilinear continuous forms corresponding to the amplitude values of the potential and kinetic energy of the system, K – stiffness matrix, M – mass matrix.

Analysis of recent research and publications shows that many works are devoted to vibrations of rods with a constant cross-section [3]. In works [4, 5], the authors consider numerical algorithms (Chebyshev and LADM-Padé methods). They are important for solving differential equations where the stiffness and mass coefficients are not constants. Banerjee [6] compared classical theories (Euler) with refined models (Rayleigh-Love, Mindlin) for vibrations. This is important for rods of large diameter, where the rotational inertia of the sections cannot be neglected. In work [7], the authors offer ready-made dynamic stiffness matrices. This allows inserting segments with a variable cross-section into the general design scheme of the structure without breaking it into thousands of small finite elements. Work [8] is devoted to the use of neural networks, the architecture of which integrates fundamental physical laws (differential equations of longitudinal vibrations). This means that the network does not simply "guess" the result, but minimizes the discrepancy of the physical equation. In [9], the author considers the method of selecting optimal cross-sections for spars and ribs, where weight is a critical factor, which often leads to the use of beams of variable cross-section. Formulas for calculating critical loads at which rod elements (stringer profiles) lose stability are analyzed in detail, which is fundamental for aircraft structures. Analysis of recent research and publications shows that the development of the theory of oscillations for rods of constant and variable cross-sections is an urgent problem that has practical applications in aircraft construction, shipbuilding and construction.

Isolation of previously unexplored parts of the general problem. There is a lack of research into the vibrations of rods of variable cross-section using effective numerical calculation methods that are implemented in software and tested on practical problems.

Research objectives. Development of a method for determining the natural forms and frequencies of oscillations of rods with both constant and variable cross-sections, which would allow optimizing design parameters to avoid resonance phenomena and increase the overall survivability of the vessel under conditions of intensive operation.

Presentation of the main material. The use of numerical approaches involves a transition from an infinite-dimensional space of admissible functions V to its finite-dimensional analogue V_h by discretization procedure. As a result, the original problem (1) is transformed into an approximate form, where for a given finite-dimensional space the following parameters should be calculated – ω, u_h, that satisfy the given conditions:

$$(Ku_h,v_h)=\omega^2(Mu_h,v_h),\ \forall v_h\in V_h\ . \tag{2}$$

When analyzing structural vibrations, the most important step is to determine the main dynamic characteristics, which allows avoiding resonance phenomena and ensuring the strength of the object. In practical engineering calculations, the main attention is paid only to the first few minimum natural frequencies and the corresponding vibration forms. Because of this, the mathematical model is reduced to an incomplete eigenvalue problem. Since the nature of this problem is nonlinear, numerical methods must be used to obtain results.

During longitudinal oscillations of the rod, the force vector coincides with its longitudinal axis. This nature of the load ensures a uniform distribution of deformations and internal stresses over the entire cross-section of the element. To describe the dynamic state of the system, the amplitude energy indicators are determined:

$$\Pi = \frac{1}{2}\int_0^l EI\left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)^2 \mathrm{d}x\,,\quad T = \frac{1}{2}\omega^2\int_0^l \rho F u^2 \mathrm{d}x\,, \tag{3}$$

where l – rod length, ρ – material density, F – cross-sectional area, E – Young's modulus.

The description of longitudinal di splacements is carried out using a linear polynomial function:

$$u(x) = u_i + \frac{u_j - u_i}{l}x\,, \tag{4}$$

here u_i, u_j – moving nodes i and j.

To describe the bending vibrations of a rod, the amplitude indices of potential and kinetic energy are determined by the formulas

$$\Pi = \frac{1}{2}\int_0^l EI\left(\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}\right)^2 \mathrm{d}x\,,\quad T = \frac{1}{2}\omega^2\int_0^l \rho F w^2 \mathrm{d}x\,. \tag{5}$$

The mathematical model of displacements is implemented by approximating them with a 3rd order polynomial:

$$w(x) = \frac{w_i}{l^3}\left(2x^3 - 3x^2 l + l^3\right) + \frac{w_j}{l^3}\left(3x^2 l - 2x^3\right) +$$
$$+\varphi_i\frac{1}{l^2}\left(xl^2 - 2x^2 l + x^3\right) + \varphi_j\frac{1}{l^3}\left(x^3 - x^2 l\right). \tag{6}$$

To solve problem (2), the iterative method of coordinate descent [10] was used, the application of which allows avoiding the difficulties associated with the formation, storage and operation with matrices of masses and stiffnesses [11].

To simulate oscillatory processes in conical or wedge-shaped rods, the initial eigenstate can be described using the mathematical expression

$$u_1(x_1) = \left(1 - \frac{x_1^2}{l^2}\right),\quad EFu_1'(0) = u_1(l) = 0\,. \tag{7}$$

For the given rods, the binomial law of variation of their physical and geometric parameters is adopted:

$$EF_1(x_1) = A(l_1 \pm x_1)^m\,,\quad \rho F(x_1) = B(l_1 \pm x_1)^n\,,\quad A = EF_1 l_1^{-m}\,,\quad B = \rho F_1 l_1^{-n}\,, \tag{8}$$

where the taper and the reduced length are respectively equal to [12]:

$$\lambda_1 = \frac{r_2}{r_1} > 1\,,\quad l_1 = l(\lambda_1 - 1).$$

The calculation of the first natural frequency is based on the energy method using the Rayleigh formula:

$$\omega_0^2 = \frac{\int_0^l EF(u')^2\,\mathrm{d}x}{\int_0^l \rho F u^2 \mathrm{d}x}\,. \tag{9}$$

Substituting (7), (8) into (9), we obtain, respectively, at $n = 1$ for the wedge and when $n = 2$ for a cone:

$$\omega_1^2 = k_1^2 \frac{E}{\rho}, \tag{10}$$

where the characteristic numbers are equal to:

$$k_1^2 = \frac{10(4l_1 + 3l)}{(5l_1 + 16l)l^2}, \quad n = 1;$$

$$k_1^2 = \frac{7\left(20l_1^2 + 30l_1 l + 12l^2\right)}{\left(8l^2 + 35l_1 l + 56l_1^2\right)l^2}, \quad n = 2. \tag{11}$$

For systems in the form of cantilevers, where the change in cross-sectional area and inertia obeys binomial laws, the analysis of bending processes takes the following form:

$$EI(x_1) = A(l_1 \pm x_1)^m, \quad \rho F(x_1) = B(l_1 \pm x_1)^n. \tag{12}$$

$$A = EI_1 l_1^{-m}, \quad B = \rho F_1 l_1^{-n}. \tag{13}$$

The mathematical description of the first eigenform can be implemented in the form of an approximate analytical dependence

$$w(x_1) = \left(1 - \frac{x_1}{l}\right)^2, \tag{14}$$

which satisfies only the kinematic boundary conditions:

$$w(0) = w'(0) = 0. \tag{15}$$

The Rayleigh formula for bending vibrations has the form

$$\omega_0^2 = \frac{\int_0^l EI(w'')^2 \, dx}{\int_0^l \rho F w^2 dx}. \tag{16}$$

Substituting (12), (14) into (16), we obtain the following formulas for the fundamental frequency:

$$\omega_1^2 = \tilde{\omega}_1^2 \left[\frac{EI_1}{\rho F_1 l^4}\right] = \tilde{\omega}_1^2 \frac{1}{\lambda_1^2 l^4}\left[\frac{EI_2}{\rho F_2}\right]. \tag{17}$$

Depending on the size of the rod taper and the ratio $\frac{EI_1}{\rho F_1}$.

In formulas (17) the frequency parameter $\tilde{\omega}_1^2$ is expressed in terms of the reduced l_1 and real l lengths of the rod as follows:

$$\tilde{\omega}_1^2 = \frac{30\left(4l_1^3 + 6l_1^2 l + 4l_1 l^2 + l^3\right)}{(6l_1 + l)l_1^2}, \quad n = 1;$$

$$\tilde{\omega}_1^2 = \frac{84\left(5l^4 + 10l_1^4 l + 10l_1^2 l + 5l_1 l^3 + l^4\right)}{\left(21l_1^2 + 7l_1 l + l^2\right)l_1^2}, \quad n = 2. \tag{18}$$

Let us evaluate the influence of the taper (wedge-shape) of the rod on the fundamental frequency in numerical examples.

Example 1.

Find the fundamental frequencies of longitudinal vibrations of a cone with a taper $\lambda_1 = 1;\ 1{,}2;\ 1{,}5;\ 2$.

According to the second formula (11), we obtain the dimensionless values of the characteristic numbers, namely $k_1 l = 1{,}581;\ 1{,}711;\ 1{,}877;\ 2{,}09$.

By substituting into formula (9), the corresponding frequencies can be obtained.

The calculation results were compared with the exact solution [6]:

$$k_1 l = 1{,}5;\ 1{,}688;\ 1{,}835;\ 2{,}03.$$

Example 2.

Determine the values of the fundamental frequencies of bending vibrations of a cantilever wedge at the following values: λ_1: $\lambda_1 = 1,\ 2,\ 4$.

Using formula (17), we calculate the value of the frequency parameter

$$\tilde{\omega}_1 = 4{,}474;\ 4{,}08;\ 4{,}21;\ 5{,}48.$$

The exact values of the frequency parameter obtained in [6] are as follows:

$$\tilde{\omega}_1 = 3{,}52;\ \text{-};\ 4{,}00;\ 4{,}61.$$

The calculation results show that with increasing wedge shape (taper), the fundamental frequency values also increase.

Conclusions. Within the framework of this study, the dynamic behavior of rod systems with variable cross-sectional geometry was analyzed. To solve the dynamics problems, a variational-grid approach was used, which allowed obtaining analytical dependencies for calculating the fundamental frequencies of both longitudinal and bending vibrations. The reliability of the results was confirmed by comparison with exact solutions. The conducted study of the influence of taper parameters on the spectrum of natural frequencies opens up opportunities for creating structures with predetermined dynamic properties. The proposed method is universal for rods of various configurations and has practical value for specialists in the field of engineering design.

References

1. Chemerys, O. M., Kolodezhnyi, V. A., Trubachev, S. I. (2017). Budivelna mekhanika mashyn [Construction mechanics of machines]. Kyiv: NTUU «KPI im. Ihoria Sikorskoho».

2. Vasylenko, M. V., Alekseichuk, O. M. (2004). Teoriia kolyvan i stiikosti rukhu [Theory of oscillations and stability of motion]. Kyiv: High school.

3. Hryhorenko, O. Ya., Pankratova, N. D. (2012). Chyselni metody rozviazannia zadach teorii kolyvan sterzhnevykh system [Numerical methods for solving problems of the theory of oscillations of rod systems]. Kyiv: Akademperiodyka.

4. Kumar, S., et al. (2025). A Non-Classical Algorithm for Vibration Analysis of Non-Uniform Beams. *International Journal for Numerical Methods in Engineering, 126 (22),* 5112–5134.

5. Singh, A., et al. (2025). Vibration Analysis of Non-uniform Beams Using Numerical Techniques. Proceedings from ICMMT-2025: *16th International Conference on Materials and Manufacturing Technologies*. (pp. 35–43). Vietnam: Ho Chi Minh City.

6. Banerjee, J. R. (2022). Dynamic stiffness method for exact longitudinal free vibration of rods and trusses using simple and advanced theories. *Applied Mathematical Modelling, 104*, 401–420.

7. Cheng, L., Yuan, J. (2022). Exact dynamic stiffness matrix for the vibration of non-uniform rods. *Mechanical Systems and Signal Processing*, *164,* 1–18.

8. Zhao, R., et al. (2022). Machine Learning-Based Vibration Analysis of Non-Uniform Rods. *Journal of Sound and Vibration*, *570*, 210–225.

9. Niu, M. C. Y. (1999). Airframe Structural Design: Practical Design Information and Data on Aircraft Structures. Hong Kong: Conmilit Press.

10. Babenko, A. Y., Boronko, O. O. & Trubachev, S. I. (2016). Zastosuvannia metoda pidvyshchennia zhorstkostei v zadachakh kolyvan bahatosharovykh konstruktsii [Application of the stiffness enhancement method in vibration problems of multilayer structures]. Proceedings from PTTIO-2016: *XVII Mizhnarodna naukovo-tekhnichna konferentsiia «Prohresyvna tekhnika, tekhnolohiia ta inzhenerna osvita»* (pp. 17-18). Kyiv: NTUU «KPI».

11. Zienkiewicz, O. C., Taylor, R. L. (2013). *The Finite Element Method: Its Basis and Fundamentals*. Oxford: Butterworth-Heinemann.

12. Bauchau, O. A., Craig, J. I. (2009). *Structural Analysis: With Applications to Aerospace Structures.* Dordrecht: Springer Science & Business Media.

Список використаних джерел

1. Чемерис, О. М., Колодежний, В. А., & Трубачев, С. І. (2017). *Будівельна механіка машин*. НТУУ «КПІ ім. Ігоря Сікорського».

2. Василенко, М. В., & Алексейчук, О. М. (2004). *Теорія коливань і стійкості руху*. Вища школа.

3. Григоренко, О. Я., & Панкратова, Н. Д. (2012). *Чисельні методи розв'язання задач теорії коливань стержневих систем*. Академперіодика.

4. Kumar, S., et al. (2025). A Non-Classical Algorithm for Vibration Analysis of Non-Uniform Beams. *International Journal for Numerical Methods in Engineering*, *126 (22)*, 5112–5134.

5. Singh, A., et al. (2025). Vibration Analysis of Non-uniform Beams Using Numerical Techniques. Proceedings from ICMMT-2025: *16th International Conference on Materials and Manufacturing Technologies*. (pp. 35–43). Vietnam: Ho Chi Minh City.

6. Banerjee, J. R. (2022). Dynamic stiffness method for exact longitudinal free vibration of rods and trusses using simple and advanced theories. *Applied Mathematical Modelling*, *104*, 401–420.

7. Cheng, L., Yuan, J. (2022). Exact dynamic stiffness matrix for the vibration of non-uniform rods. *Mechanical Systems and Signal Processing*, *164,* 1–18.

8. Zhao, R., et al. (2022). Machine Learning-Based Vibration Analysis of Non-Uniform Rods. *Journal of Sound and Vibration*, *570*, 210–225.

9. Niu, M. C. Y. (1999). *Airframe structural design: Practical design information and data on aircraft structures*. Conmilit Press.

10. Бабенко, А. Є., Боронко, О. О., & Трубачев, С. І. (2016). Застосування метода підвищення жорсткостей в задачах коливань багатошарових конструкцій. У *Прогресивна техніка, технологія та інженерна освіта (ПТТІО-2016)* (с. 17–18). НТУУ «КПІ».

11. Zienkiewicz, O. C., & Taylor, R. L. (2013). *The Finite Element Method: Its Basis and Fundamentals.* Oxford: Butterworth-Heinemann.

12. Bauchau, O. A, & Craig, J. I. (2009). *Structural Analysis: With Applications to Aerospace Structures.* Dordrecht: Springer Science & Business Media.

Дата першого надходження статті до видання: 03.03.2026
Дата прийняття статті до друку після рецензування: 24.03.2026

УДК 539.3

Сергій Трубачев[1], Валерій Колодежний[2]

[1]кандидат технічних наук, доцент, доцент кафедри динаміки і міцності машин та опору матеріалів
Національний технічний університет України
«Київський політехнічний інститут імені Ігоря Сікорського» (Київ, Україна)
E-mail: strubachev1@ukr.net. **ORCID**: https://orcid.org/0000-0002-7349-9426
ResearcherID: AAK-6228-2020. **Scopus Author ID**: 16437921100

[2]старший викладач кафедри динаміки і міцності машин та опору матеріалів
Національний технічний університет України
«Київський політехнічний інститут імені Ігоря Сікорського» (Київ, Україна)
E-mail: bokshin@ukr.net. **ORCID:** https://orcid.org/0000-0002-1722-1390. **ResearcherID:** l-CtlTMAAAAJ

ВИЗНАЧЕННЯ ДИНАМІЧНИХ ХАРАКТЕРИСТИК СТРИЖНІВ КОНСТРУКЦІЙ СУДНОБУДІВНИЦТВА

Сучасне суднобудування розвивається в напрямку збільшення тоннажу суден та потужності їхніх енергетичних установок, що супроводжується активним використанням полегшених конструкційних матеріалів. У таких умовах критично гостро постає проблема забезпечення вібраційної міцності та надійності корпусних конструкцій, оскільки неконтрольовані коливання стрижневих елементів — базових компонентів набору корпусу, щогл та валопроводів — призводять до втомного руйнування, зниження точності навігаційного обладнання та погіршення умов для екіпажу.

Традиційні аналітичні методи мають обмежене застосування для реальних об'єктів зі складною геометрією та змінною жорсткістю, а чисельні методи потребують ефективних алгоритмів для точного прогнозування динамічних характеристик на етапі проектування.

У роботі було розроблено та науково обґрунтовано комплексну методику визначення власних форм і частот коливань стрижневих елементів як постійного, так і змінного перерізу із використанням варіаційно-сіткового підходу.

Також набув подальшого розвитку ітераційний метод покоординатного спуску, застосування якого в задачах динаміки стрижнів дозволяє уникнути обчислювальних труднощів, пов'язаних із формуванням та оперуванням з глобальними матрицями мас і жорсткості. Крім того, застосування метода покоординатного спуску в задачах коливань дозволяє вирішувати задачі великої розмірності, використовуючи тільки оперативну пам'ять ПЕОМ. Помилки округлення не накопичуються.

Отримано аналітичні вирази для визначення основних власних частот поздовжніх та згинальних коливань для стрижнів, фізико-геометричні характеристики яких змінюються за біноміальними законами. Досліджено характер впливу параметрів конусності та клиновидності на спектр власних частот. Розроблений підхід апробовано на прикладах розрахунку консольних стрижнів, що дозволяє проектувати суднової конструкції із заздалегідь визначеними динамічними властивостями для уникнення резонансних явищ.

Ключові слова: *коливання; динамічні характеристики; стрижневі елементи; змінний переріз; варіаційно-сітковий підхід; метод покоординатного спуску.*

Бібл.: 7.

Trubachev, S., & Kolodezhnyi, V. (2026) Determination of dynamic characteristics of rods of shipbuilding structures. *Technical sciences and technologies*, (2(44)), 131-137. DOI: https://doi.org/10.25140/2411-5363-2026-2(44)-131-137.